West Spring Secondary School
3E Elementary Mathematics: Assignment 34
Trigonometry

Name: _____ ( ) **Class:** _____ **Date:** _____

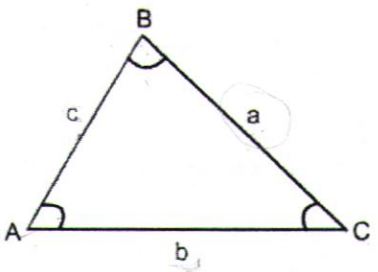
Overview

This worksheet covers the following:

1. Sine rule
2. Cosine rule

Sine Rule

Given a triangle,



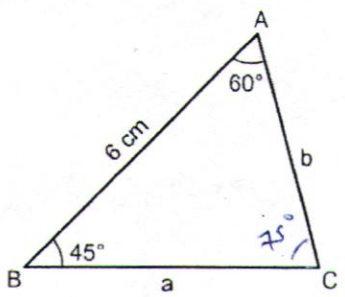
Sine rule: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

or $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$\frac{6}{2} = \frac{3}{1} = \frac{9}{3}$

Example

1. **Find the missing angle and sides**



(Answers: $\angle C = 75°, a = 5.38cm, b = 4.39cm$)

$\angle C = 180° - 60° - 45°$

$= 75°$

$\frac{\sin 75°}{6} = \frac{\sin 60°}{a}$

$a \sin 75° = 6 \sin 60°$

$a = \frac{6 \sin 60°}{\sin 75°}$

$a = 5.3794$

$a = 5.38$ cm (3 sig. fig.)

$\frac{\sin 75°}{6} = \frac{\sin 45°}{b}$

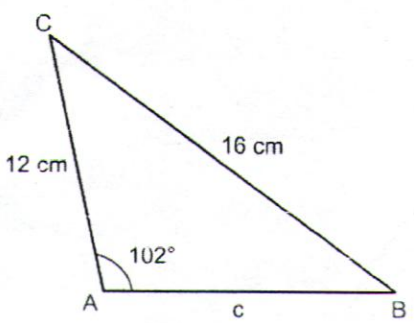
$b \sin 75° = 6 \sin 45°$

$b = \frac{6 \sin 45°}{\sin 75°}$

$b = 4.3923$

$b = 4.39$ cm (3 sig. fig.)

2. **Find the missing angles and side**



(Answers: $\angle B = 47.2°, \angle C = 30.8°, c = 8.38cm$)

$\frac{\sin 102°}{16} = \frac{\sin \hat{B}}{12}$

$12 \sin 102° = 16 \sin \hat{B}$

$16 \sin \hat{B} = 12 \sin 102°$

$\sin \hat{B} = \frac{12 \sin 102°}{16}$

$\hat{B} = 47.189°$

$\hat{B} = 47.2°$ (1 dec. pl.)

$\angle C = 180° - 102° - 47.189°$

$= 30.811°$

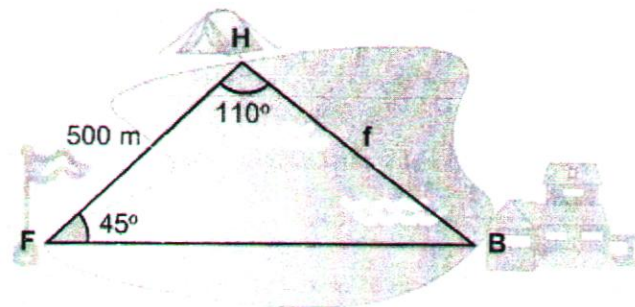
$= 30.8°$ (1 dec. pl.)

$\frac{\sin 102°}{16} = \frac{\sin 30.811°}{c}$

$c = 8.3784 = 8.38$ cm (3 sig. fig.)

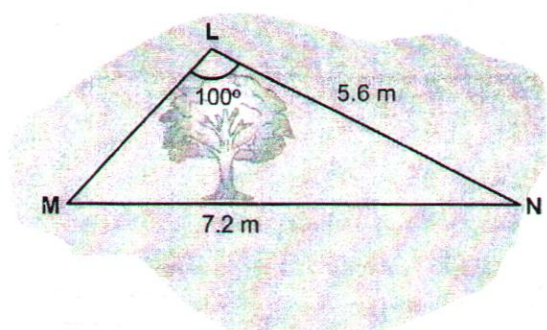
Practice

1. Some campers want to find the distance f across a lake as shown in the diagram. The distance from the flagpole (F) to the edge of the headquarters (H) and the angles are measured and indicated. Find the distance f across the lake.



(Answers: $f = 837m$)

2. ACE Secondary school wants to preserve a Tembusu tree that is at the heart of its garden. It decides to build a wooden fence, △LMN, around it.
Given that $M\hat{L}N$ = 100°, LN = 5.6 cm and MN = 7.2 cm, solve △LMN.



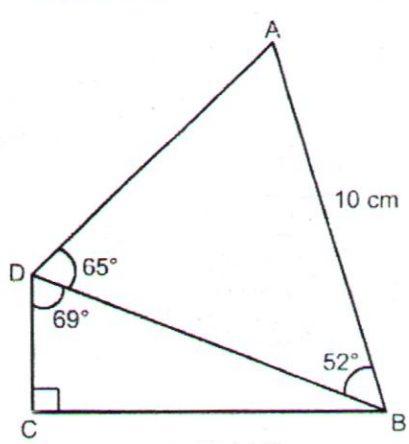
(Answers: $\angle M = 50.0°, \angle N = 30°, ML = 3.66m$)

3. **Calculate from the diagram below:**

i) BD

ii) BC

(Answers: $i) BD = 9.83cm$)
$ii) BC = 9.18cm$



$a^2 = b^2 + c^2 - 2bc\cos\hat{A}$ $\quad b^2 = a^2 + c^2 - 2ac\cos\hat{B}$

$c^2 = a^2 + b^2 - 2ab\cos\hat{C}$

Cosine Rule

The Cosine Rule states that:

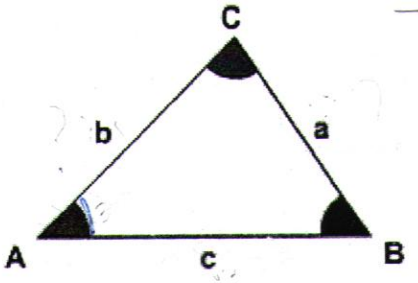
finding length.

In any ΔABC,

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$



where a is the side opposite $\hat{A}$, b is the side opposite $\hat{B}$ and c is the side opposite $\hat{C}$.

When the formula is rearranged, the cosine of the angle in terms of the lengths of the two sides of the triangle is obtained.

finding angles.
(must know all 3 sides)

$$a^2 = b^2 + c^2 - 2bc\cos A \quad \Rightarrow \quad \cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$b^2 = a^2 + c^2 - 2ac\cos B \quad \Rightarrow \quad \cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$c^2 = a^2 + b^2 - 2ab\cos C \quad \Rightarrow \quad \cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

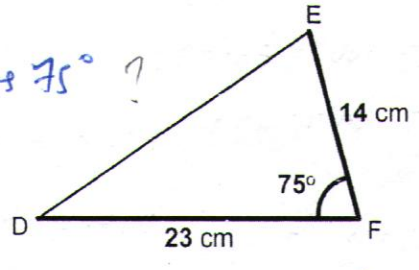
Useful Tips

Cosine Rule: To be used when you are given

1. Any two sides and one **included** angle, find the remaining side ➔ $a^2 = b^2 + c^2 - 2bc\cos A$
2. All 3 sides, find an angle ➔ $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

Example

1. In ΔDEF, EF = 14 cm, DF = 23 cm and $\hat{F} = 75°$. Find DE, giving your answer correct to 2 decimal places.



$DE^2 = (14)^2 + (23)^2 - 2(14)(23)\cos 75°$?

$DE = 23.628$

$DE = 23.63\text{cm}$

~~(3 sig. fig.)~~

2 dec. pl.

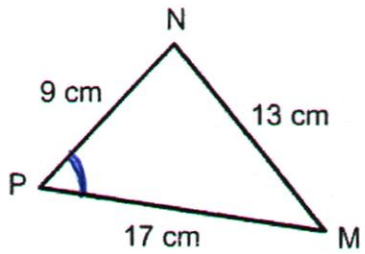
$\cos\hat{A} = \dfrac{(b)^2 + (c)^2 - (a)^2}{2(b)(c)}$

(Answers: $DE = 23.6cm$)

$\cos\hat{B} = \dfrac{(a)^2 + (c)^2 - (b)^2}{2(a)(c)}$

2. If $p = 13cm$, $m = 9cm$ and $n = 17cm$.

a) Calculate $\hat{P}$ of ΔPMN.
b) Find the size of the largest angle



(a) $\cos \hat{P} = \dfrac{(9)^2 + (17)^2 - (13)^2}{2(9)(17)}$

$\hat{P} = 48.938^\circ$

$\hat{P} = 48.9^\circ$ (1 dec. pl.)

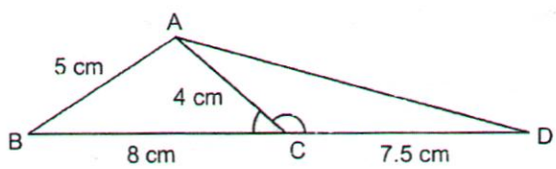
(b) $\cos \hat{N} = \dfrac{(9)^2 + (13)^2 - (17)^2}{2(9)(13)}$

$\hat{N} = 99.594^\circ$

$\hat{N} = 99.6^\circ$ (1 dec. pl.)

(Answers: $\angle P = 48.9^\circ, \angle N = 99.6^\circ$)

3. In the figure, find AD, give your answer correct to 2 decimal places.



$\cos \hat{ACB} = \dfrac{(4)^2 + (8)^2 - (5)^2}{2(4)(8)}$

$\hat{ACB} = 30.753^\circ$

$\hat{ACD} = 180^\circ - 30.753^\circ$

$= 149.247^\circ$

$AD^2 = (7.5)^2 + (4)^2 - 2(7.5)(4)\cos 149.247^\circ$

$AD = 11.127$

$AD = 11.13\ cm$

(~~3 sig. fig~~)

2 dec. pl.

(Answers: $AD = 11.1cm$)

Practice

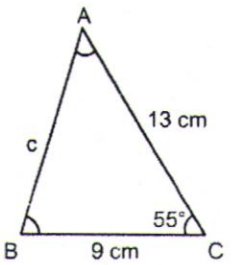
1. **Find the length of side c**

Cosine Rule:

$a^2 = b^2 + c^2 - 2bc \cos A$

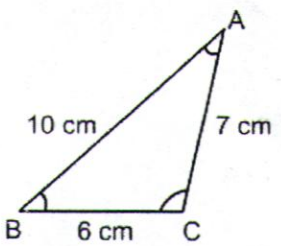
$b^2 = a^2 + c^2 - 2ac \cos B$

$c^2 = a^2 + b^2 - 2ab \cos C$



(Answers: $c = 10.8cm$)

2. **Find the size of the largest angle**

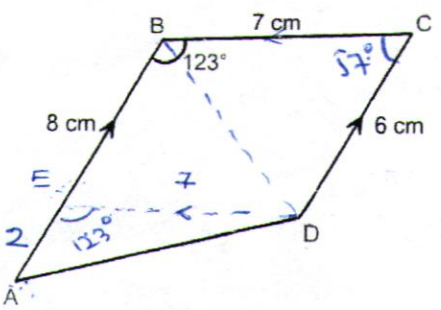


(Answers: $\angle C = 100.3°$)

3. **In the diagram, ABCD is a trapezium in which AB and DC are parallel.**

Calculate i) BD

ii) AD

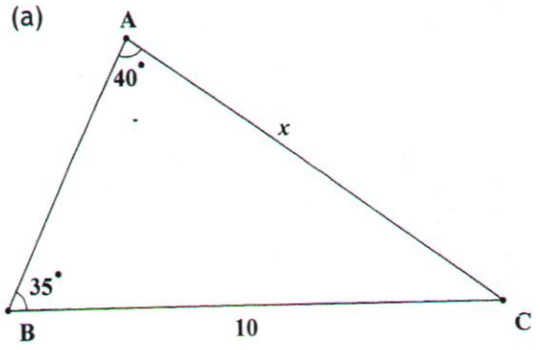


(Answers: $i) BD = 6.27cm, ii) AD = 8.26cm$)

Homework

1. Find the unknown lengths x in each of the following:
(Answers: $a) x = 8.92, b) x = 20.4$)

(a)

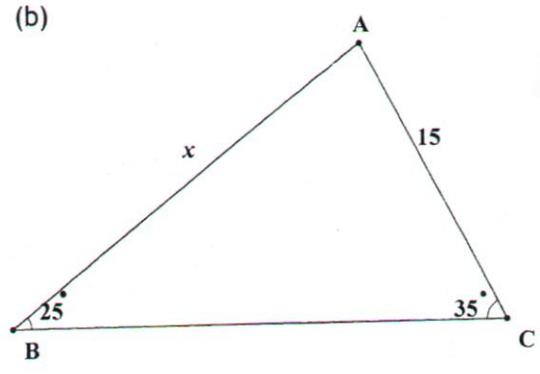


$$\frac{x}{\sin 35^\circ} = \frac{10}{\sin 40^\circ}$$

$$x = 8.9232$$

$$x = 8.92 \text{ (3 sig. fig.)}$$

(b)



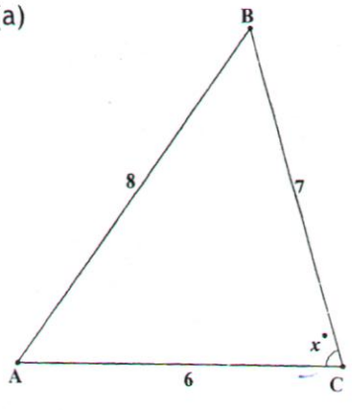
$$\frac{x}{\sin 35^\circ} = \frac{15}{\sin 25^\circ}$$

$$x = 20.357$$

$$x = 20.4 \text{ (3 sig. fig.)}$$

2. Find x in each of the following:
(Answers: $a) \angle x = 75.5^\circ, b) x = 6.82$)

(a)

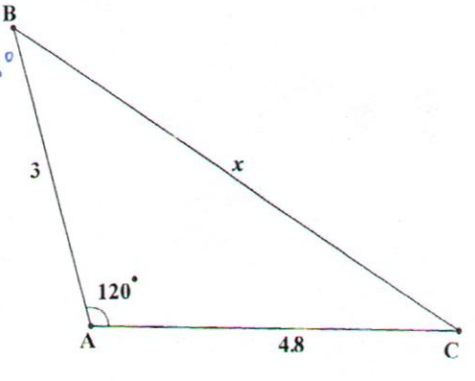


$$\cos \hat{x} = \frac{6^2 + 7^2 - 8^2}{2(6)(7)}$$

$$x = 75.522^\circ$$

$$x = 75.5^\circ \text{ (1 dec. pl.)}$$

(b)



$$x^2 = 3^2 + 4.8^2 - 2(3)(4.8)\cos 120^\circ$$

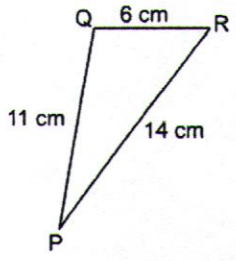
$$x^2 = 46.44$$

$$x = \sqrt{46.44}$$

$$x = 6.814$$

$$x = 6.81 \text{ (3 sig. fig.)}$$

3. i) Find the size of angle Q

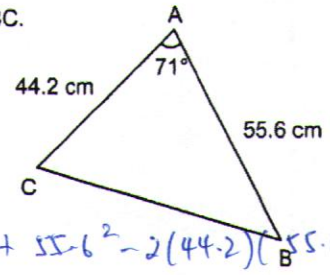


$\cos \hat{Q} = \frac{11^2 + 6^2 - 14^2}{2(6)(11)}$

$\hat{Q} = 107.184^\circ$

$\hat{Q} = 107.2^\circ$ (1 dec. pl.)

ii) Find the length of BC.



$BC^2 = 44.2^2 + 55.6^2 - 2(44.2)(55.6)\cos 71^\circ$

$BC^2 = 3444.8194$

$BC = 58.6925$

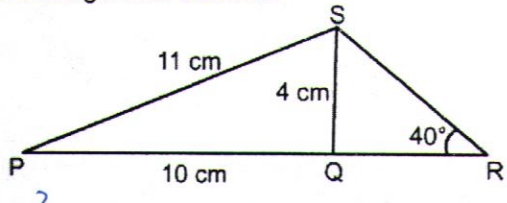
$BC = 58.7$ cm, (3 sig. fig.)

(Answers: *i)*$\angle Q = 107.2^\circ$, *ii)*$BC = 58.7cm$)

4. In the diagram, PQR is a straight line. Calculate

(i) ∠SQR

(ii) QR



(i) $\cos S\hat{Q}P = \frac{4^2 + 10^2 - 11^2}{2(4)(10)}$

$\cos S\hat{Q}P = -0.0625$

$S\hat{Q}P = 93.583^\circ$

$S\hat{Q}R = 180^\circ - 93.583^\circ$

$= 86.417^\circ$

$= 86.4^\circ$ (1 dec. pl.)

(ii) $Q\hat{S}R = 180^\circ - 40^\circ - 86.417^\circ$

$= 53.583^\circ$

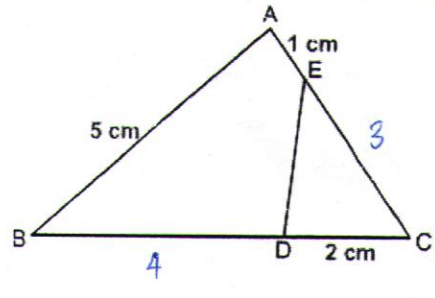
$\frac{QR}{\sin 53.583^\circ} = \frac{4}{\sin 40^\circ}$

$QR = 5.0076$

$QR = 5.01$ cm (3 sig. fig.)

(Answers: *i)*$\angle SQR = 86.4^\circ$, *ii)*$QR = 5.00cm$)

5. In the figure, CD = 2cm, AE = 1cm, AB = 5cm, BD = 2CD and CE = 3AE. Find DE.
(Hint: find angle ACB first) (Answers: $DE = 2.50cm$)



$\cos A\hat{C}B = \frac{4^2 + 6^2 - 5^2}{2(4)(6)}$

$\cos A\hat{C}B = 0.5625$

$A\hat{C}B = 55.7711^\circ$

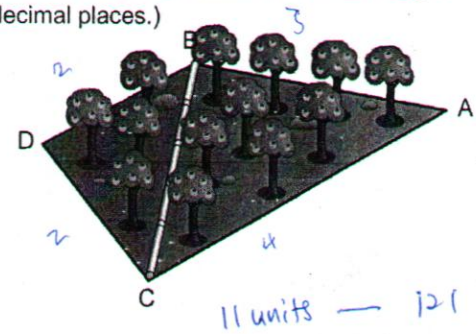
$DE^2 = 3^2 + 2^2 - 2(3)(2)\cos 55.771^\circ$

$DE = 2.4999$

$DE = 2.50$ cm

(3 sig. fig.)

6. A trapezoid field, which has a perimeter of 121 m, has sides in the ratio 2 : 2 : 4 : 3. An irrigation pipe is to be installed as shown. If the angle between two of its longer sides is 40°, how long is the irrigation pipe? (Give your answer correct to 2 decimal places.)



$BC^2 = 33^2 + 44^2 - 2(33)(44)\cos 40°$

$BC = 28.2914$

$BC = 28.29\ m$

(2 dec. pl.)

11 units — 121

3 units — 33 m

4 units — 44 m

(Answers: 28.29*m*)

Summary

Illustrate the sine rule and cosine rule with appropriate diagrams using the space below:

Sine Rule	**Cosine Rule**

1 To find a woman you need Time and Money therefore:

Woman = Time x Money

2 "Time is money" so

Time = Money

3 Therefore

Woman = Money x Money

Woman = (Money)²

4 "Money is the root of all problems"

Money = √Problems

5 Therefore

Woman = (√Problems)²

Woman = Problems

A+

My Reflection:

Deadline: Jun 2011