West Spring Secondary School
3E Elementary Mathematics Assignment 32: Trigonometry

Name: ________________________ (   ) Class: ___________ Date: _________

Recap

Previously, we learned about:

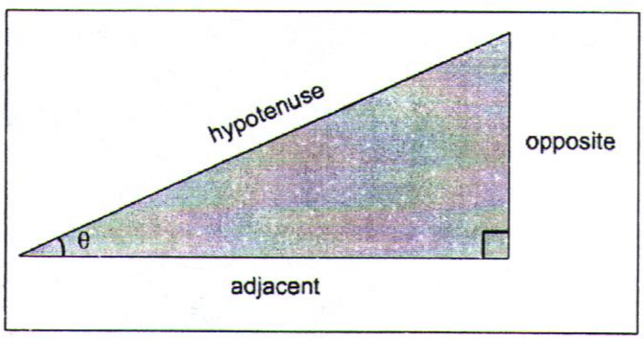
1. Trigonometric ratios
2. Use of trigonometric ratios to find angles and sides of a triangle

Overview

This worksheet covers the following:

1. Trigonometric ratios for obtuse angles
2. Area of a triangle

Introduction



The Three Trigonometric Ratios

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

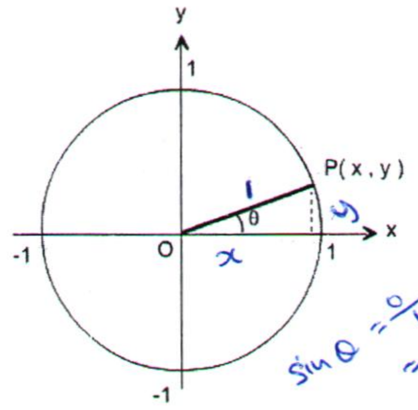
Previously we learned the definition of three basic trigo ratios. These ratios were defined for acute angles.

Trigo ratios can be defined for obtuse angles as well. Let's see how we do that.

Trigo Ratios for Obtuse Angles

To defined trigo ratios for obtuse angles, we use the concept of coordinate geometry.

When θ is acute,



Consider a unit circle (radius = 1 unit).

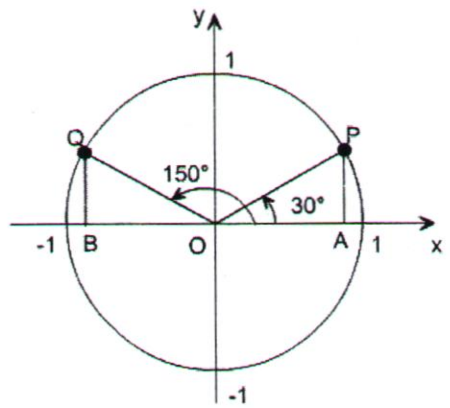
The cosine of an angle is the ________________ of the point P on a unit circle.

The sine of an angle is the ________________ of the point P on a unit circle.

$\sin\theta = o/h$
$= y$

$\cos\theta = a/h$
$= x$

When θ is obtuse,



AP = ________

Hence, $\sin 30^o =$ 0.5

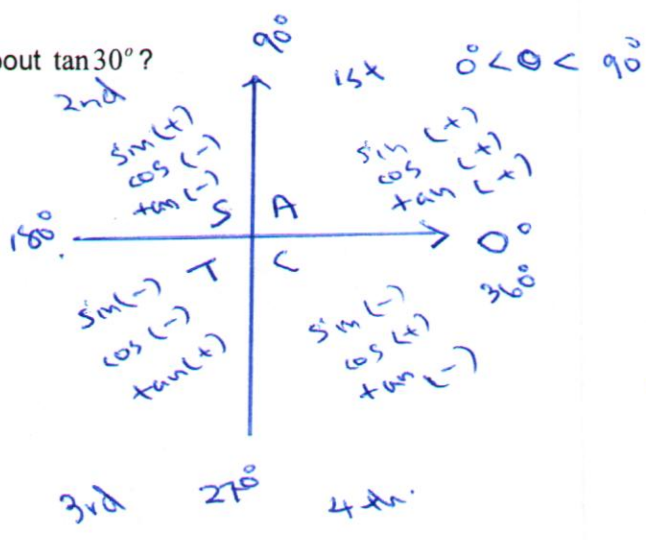
In general, $\sin\theta = \sin(180^\circ - \theta)$

OA = ________

Hence, $\cos 30^o =$ 0.866

In general, $\cos\theta = -\cos(180^\circ - \theta)$

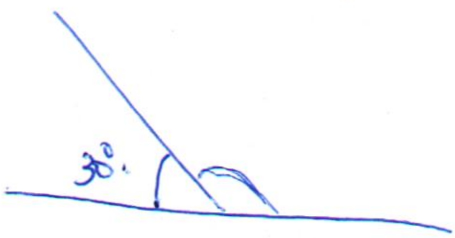
What about $\tan 30^o$?



$\sin\theta = \sin(180^\circ - \theta)$

$\cos\theta = -\cos(180^\circ - \theta)$

$\tan\theta = -\tan(180^\circ - \theta)$



Examples:

1. Solve sin x = 0.76 for values between 0° and 180°.

$\sin x = 0.76$

$x = 49.46°$

$\sin x = \sin(180° - x)$

$\sin 49.46° = \sin(180° - 49.46°)$ $\therefore \theta = 49.5°$ or $\theta = 130.5°$

$\sin 49.46° = \sin 130.54°$

2. Given cos 32° = 0.8, find the value of cos 148°.

$\cos 148° = -\cos(180° - 148°)$

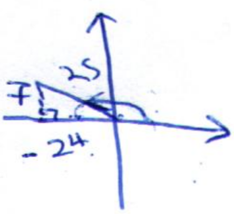
$\cos 148° = -\cos 32°$

$= -0.8$

3. If A is an obtuse angle and cos A = $-\frac{24}{25}$, find without using a table or calculator, the values of:

a, h

   i) sin A
   ii) tan A



i) $\sin A = \frac{o}{h}$

$= \frac{7}{25}$

$\sin\theta = \sin(180° - \theta)$

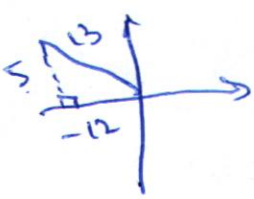
$\sin 16.26° = \sin 134.74$

ii) $\tan A = \frac{o}{a}$

$= -\frac{7}{24}$

4. If tan B = $-\frac{5}{12}$, such that $0^o < B < 180^o$, find without using a table or calculator, the values of

   i) sin B
   ii) cos B



i) $\sin B = \frac{o}{h}$

$= \frac{5}{13}$

ii) $\cos B = \frac{a}{h}$

$= \frac{-12}{13}$

Answers
1) x = 49.5°, 130.5° 2) -0.8 3i) 7/25 ii) -7/24 4i) 5/13 ii) -12/13

Practice

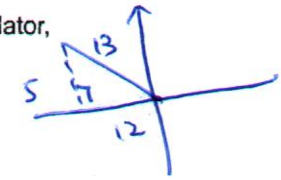
1. Solve sin x = 0.23 for values between 0° and 180°. Give your answers to 1 d.p.

2. Explain why the equation sin x = 1 has only one solution between 0° and 180°. Write down the solution.

3. Express tan 153° as a tan ratio of an acute angle and hence find its value to 2 d.p.

4. If A is an obtuse angle and sin A = $\frac{5}{13}$, find, without using a table or calculator, the values of (i) cos A

   (ii) tan A.



5. If cos B = $-\frac{8}{17}$, such that 0° < B < 180°, find, without using a table or calculator, the values of (i) sin B

   (ii) tan B.

i) $\cos A = \frac{a}{h} = \frac{-12}{13}$

(ii) $\tan A = \frac{o}{a} = -\frac{5}{12}$

1. $\sin x = 0.23$

$x = \cancel{10.3}\ 13.29$

$\sin \theta = \sin(180° - \theta)$

$\sin 13.29° = \sin(180° - 13.29°)$

$\sin 13.29° = \sin 166.71$

$\therefore \theta = 13.3°$ or $\theta = 166.7°$ (1 dec. pl).

3. $\tan 153° = -\tan(180° - 153°)$

$\tan 153° = -\tan 27°$

$= -0.509$

$= -0.51$ (2 dec. pl.)

2. $\sin x = 1$

$x = 90°$

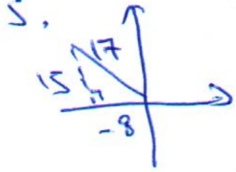
$\sin \theta = \sin(180° - \theta)$

$\sin 90° = \sin(180° - 90°)$

$\sin 90° = \sin 90°$

$\therefore \theta = 90°$

5.



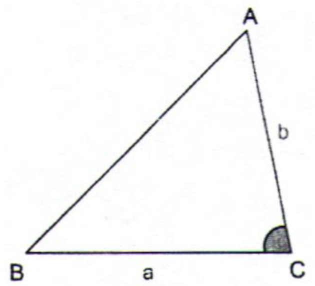
i) $\sin B = \frac{15}{17}$

ii) $\tan B = -\frac{15}{8}$

Answers
1) x = 13.3°, 166.7°   4) -0.51   4i) -12/13 ii) -5/12   5i) 15/17 ii) -15/8

Area of Triangle

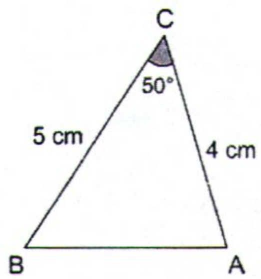
Given BC = a, AC = b and the included angle C, find the area of triangle ABC



Area of Triangle = $\frac{1}{2} ab \sin \theta$

Practice:

1. **Find the area of this triangle**

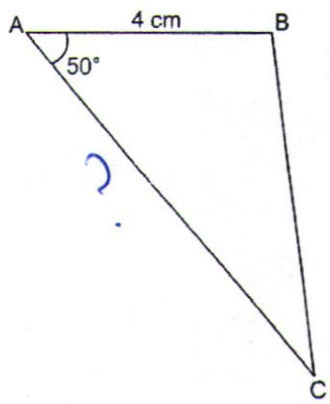


$$\text{Area} = \frac{1}{2}(5)(4)\sin 50^\circ$$

$$= 7.6604$$

$$= 7.66 \text{ cm}^2 \text{ (3 sig. fig.)}$$

2. **Find length AC if area of ΔABC = 11 cm^2.**



$$\frac{1}{2} AC(4)\sin 50^\circ = 11$$

$$2\, AC \sin 50^\circ = 11$$

$$AC = \frac{11}{2\sin 50^\circ}$$

$$= 7.1797$$

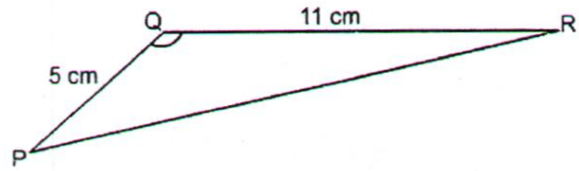
$$= 7.18 \text{ cm}$$

(3 sig. fig.)

Answers
1) 7.66 cm^2 2) 7.18 cm

Homework

1. **Find $\angle Q$ if area of $\Delta PQR = 18$ cm^2.**



$\frac{1}{2}(5)(11)\sin Q = 18$

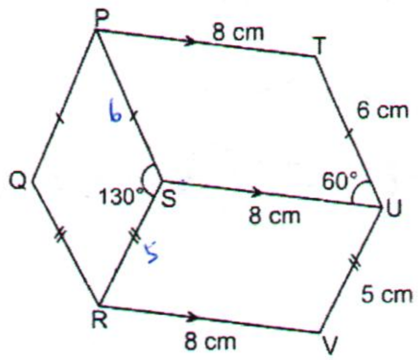
$\frac{55 \sin Q}{2} = 18$

$\sin Q = \frac{36}{55}$

$Q = 40.885^\circ$

$Q = 40.9^\circ$ (correct to 1 dec. pl.)

2. **Find the area of the figure given.**



$\angle PSU = 180^\circ - 60^\circ$ (int. $\angle$s)

$= 120^\circ$

$\angle RSU = 360^\circ - 120^\circ - 130^\circ$ ($\angle$s at a pt.)

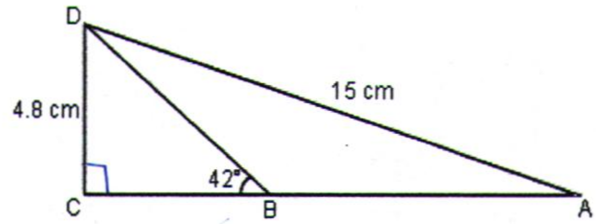
$= 110^\circ$

Area of figure $= 2(\frac{1}{2})(6)(5)\sin 130^\circ + 2(\frac{1}{2})(6)(8)\sin 120^\circ + 2(\frac{1}{2})(5)(8)\sin 110^\circ$

$= 102.138$

$= 102$ cm^2 (correct to 3 sig. fig.)

3. In the figure given, AD = 15 cm, DC = 4.8 cm, $\angle DBC = 42^\circ$.
Calculate
a. $\angle ADC$
b. Area of ΔADC
c. Length of BC



(a) $\cos \hat{ADC} = \frac{4.8}{15}$

$\hat{ADC} = 71.337$

$\hat{ADC} = 71.3^\circ$ (correct to 1 dec. pl.)

(b) Area of $\Delta ADC = \frac{1}{2}(4.8)(15)\sin 71.337^\circ$

$= 34.107$

$= 34.1 \text{ cm}^2$ (3 sig. fig.)

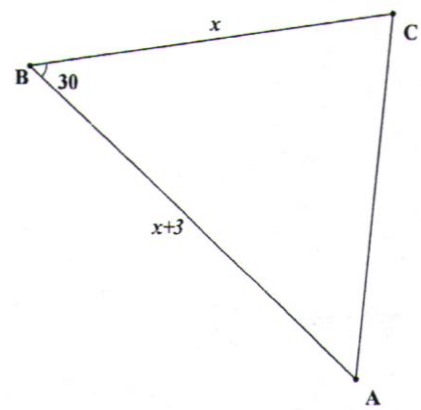
c) $\tan 42^\circ = \frac{4.8}{BC}$

$BC = 5.3309$

$BC = 5.33$ cm

(3 sig. fig.)

4. In the diagram, AB = $(x + 3)$ cm, bC = x cm and $\angle ABC = 30^\circ$. If the area of $\Delta ABC = 27 \text{ cm}^2$, find the value of x.



$\frac{1}{2}(x)(x+3)\sin 30^\circ = 27$

$\frac{x(x+3)}{4} = 27$

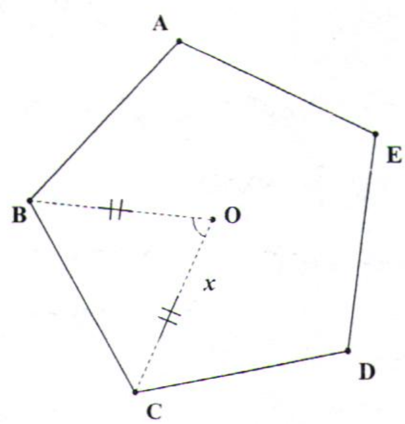
$x^2 + 3x - 108 = 0$

$(x+12)(x-9) = 0$

$x = -12$ or $x = 9$

(N.A.)

5. ABCDE is a regular pentagon made up of 5 isosceles triangles with centre at O.
a. Calculate the angle BOC subtended by each small triangle.
b. Given that the area of the whole pentagon is 400 cm², find the length x.



a) $\hat{BOC} = 360^\circ \div 5$

$= 72^\circ$

b) Area of $\Delta BOC = 400 \div 5$

$= 80 \text{ cm}^2$

$\frac{1}{2}(x)(x)\sin 72^\circ = 80$

$x^2 = \frac{160}{\sin 72^\circ}$

$x = 12.97$

$x = 13.0$ cm

(3 sig. fig.)

6. Find the possible values of θ for $0° \le \theta \le 180°$ in each of the following:
(a) $\sin\theta = 0.4$ (b) $\cos\theta = 0.5$

$\theta = 23.578$

$\sin 23.578° = \sin(180° - 23.578°)$

$\sin 23.578° = \sin 156.422°$

$\theta = 60°$

→ θ = 23.6° and 156.4° → θ = 60°

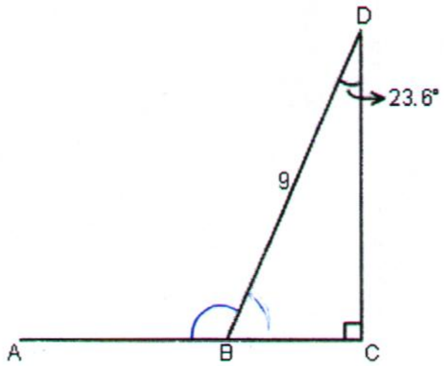
7. In the diagram, ABC is a straight line, DB = 9cm, ∠ BCD = 90° and ∠BDC = 23.6°.
Calculate

a. BC

b. ∠CBD

c. cos ∠CBD

d. cos ∠ABD



(a) $\sin 23.6° = \frac{BC}{9}$

$BC = 9\sin 23.6°$

$= 3.603$

$= 3.60$ (3 sig. fig.)

(b) $\angle CBD = 180° - 23.6° - 90°$

$= 66.4°$

(c) $\cos 66.4° = 0.4003$

$= 0.400$

(d) $\cos A\hat{B}D = -\cos(180° - A\hat{B}D)$

$\cos A\hat{B}D = -\cos C\hat{B}D$

$= -0.400$

Answers

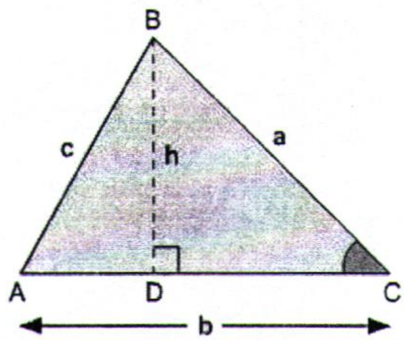
1) 40.9° 2) 102 cm^2 3a) 71.3° b) 34.1 cm^2 c) 5.33 cm 4) x = 9, -12(rejected)
5a) 72° b) 13.0 cm 6a) 23.6°, 156.4° b) 60° 7a) 3.60 cm b) 66.4° c) 0.4 d) -0.4

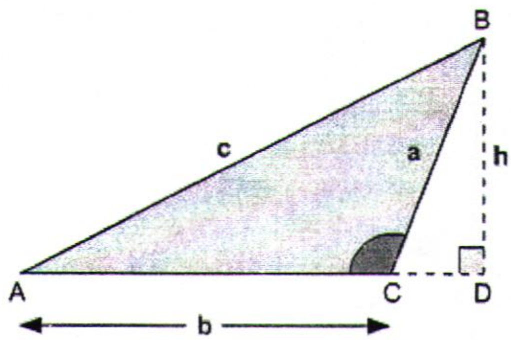
Summary

Area of Triangle

The area of a triangle can be found if two of its sides and the included angle (acute or obtuse) are given. An included angle is an angle between two given sides of a triangle.

$$\begin{aligned} Area \text{ of a } \Delta &= \frac{1}{2}ab\sin C \\ &= \frac{1}{2}bc\sin A \\ &= \frac{1}{2}ac\sin B \end{aligned}$$





where a is the side opposite $\hat{A}$, b is the side opposite $\hat{B}$ and c is the side opposite $\hat{C}$.

Solving equation by one Blondie:

$$\frac{1}{n}\sin x = ?$$

$$\frac{1}{\cancel{n}}\mathrm{si}\cancel{n}\, x =$$

$$six = 6$$

Deadline: June 2011

My Reflection: