



Name: \_\_\_\_\_ ( ) Class: \_\_\_\_\_ Date: \_\_\_\_\_

### Overview

This worksheet covers the following:

1. Introduction to trigonometric ratios
2. Use trigonometric ratios to find unknown sides or angles

### Introduction

(www.ace-learning.com.sg)

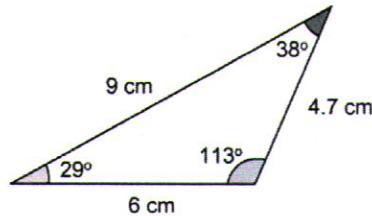
#### What is Trigonometry?

Trigonometry comes from the Greek words

*trigonon* meaning three angles

and

*metro* meaning measure.



It is a specialized area of geometry concerned with the relationships between the sides and angles of triangles.

Trigonometry helps us to make indirect measurements. This is very useful because we may not always be able to make direct measurements due to physical constraints or safety considerations.

Who invented trigonometry?

Presently, navigation and land surveying are heavily dependent on Trigonometry.

Without trigonometry, we would not be able to launch trajectories to send men to the moon!

The ancient **Egyptians**, **Babylonians** and Indus Valley civilizations were the earliest people to use Trigonometry.



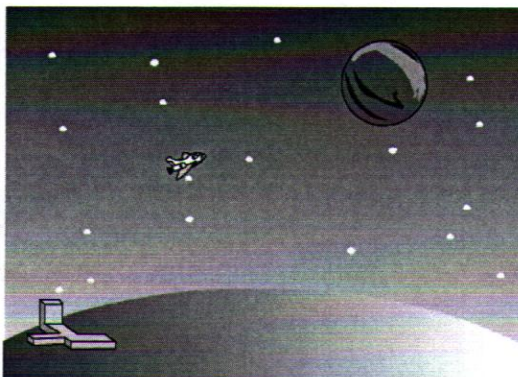
Egyptians



Babylonians

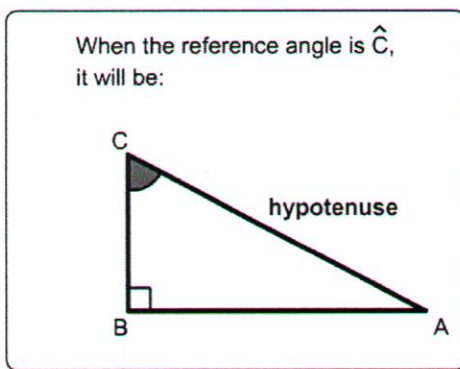
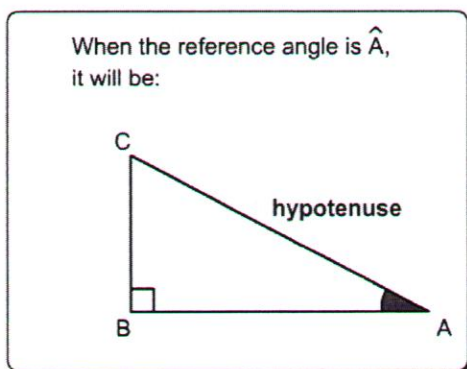


Indus Valley civilizations



In our previous lesson on the Pythagoras' Theorem, you would recall that in a right-angled triangle, the side opposite the right angle is called the **hypotenuse**. It is the longest side of a right-angled triangle.

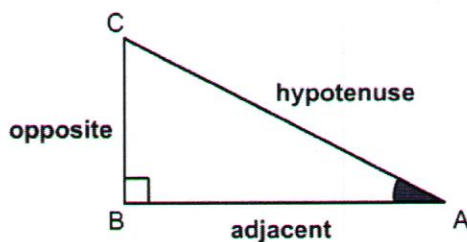
Let's take a look at how the other 2 sides are named with respect to the reference angle.



**Opposite** : Length of side opposite to the reference angle  
**Adjacent** : Length of side adjacent to the reference angle

### The 3 Basic Trigo Ratios

The 3 basic Trigonometric Ratios are namely, **Sine Ratio**, **Cosine Ratio** and **Tangent Ratio**.



Each ratio is defined as the ratio of two particular sides of a triangle.

$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin A = \frac{O}{H}$$

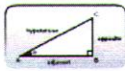
$$\cos A = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos A = \frac{A}{H}$$

$$\tan A = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan A = \frac{O}{A}$$

Summary and Mnemonics



$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}} ; \cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}} ; \tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

In short  $\Rightarrow \sin \theta = \frac{\text{opp}}{\text{hyp}} ; \cos \theta = \frac{\text{adj}}{\text{hyp}} ; \tan \theta = \frac{\text{opp}}{\text{adj}}$

Abbreviated  $\Rightarrow S = \frac{O}{H} ; C = \frac{A}{H} ; T = \frac{O}{A}$

Use mnemonics  $\Rightarrow$  **SOH** ; **CAH** ; **TOA**

It is easier to remember it as



"TOA CAH SOH" (Big, Foot, Aunty in the Chinese Hokkien dialect)

Practice

1. For the right-angled triangle PQR, find

- (i)  $\sin P$       (ii)  $\cos P$       (iii)  $\tan P$   
 (iv)  $\sin R$       (v)  $\cos R$       (vi)  $\tan R$

(i)  $\sin P = \frac{3}{5}$

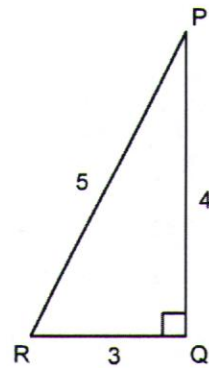
(iv)  $\sin R = \frac{4}{5}$

(ii)  $\cos P = \frac{4}{5}$

(v)  $\cos R = \frac{3}{5}$

(iii)  $\tan P = \frac{3}{4}$

(vi)  $\tan R = \frac{4}{3}$

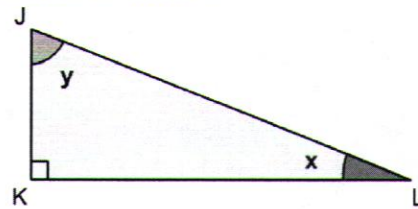


2. JKL is a right-angled triangle.

Name the sides which are opposite, adjacent and hypotenuse with respect to

- (a) x  
 (b) y

- (a) JK (opposite)      KL (opposite)  
 KL (adjacent)      JK (adjacent)  
 JL (hypotenuse)      JL (hypotenuse)



3. Given that  $\Delta PQR$  is a right-angled triangle, find

- (a)  $\sin Q$       (c)  $\tan Q$       (e)  $\cos R$   
 (b)  $\cos Q$       (d)  $\sin R$       (f)  $\tan R$   
 in term of x, y and z.

(a)  $\sin Q = \frac{y}{z}$

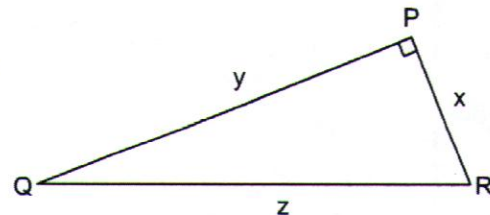
(d)  $\sin R = \frac{y}{z}$

(b)  $\cos Q = \frac{x}{z}$

(e)  $\cos R = \frac{x}{z}$

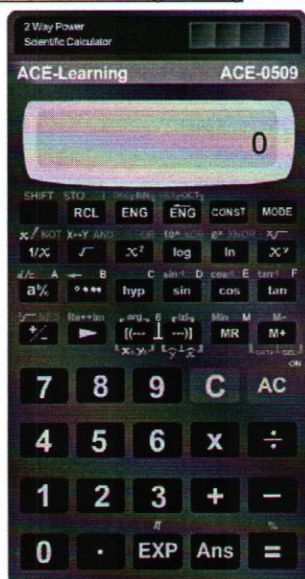
(c)  $\tan Q = \frac{y}{x}$

(f)  $\tan R = \frac{y}{x}$



## Use of Calculator

([www.ace-learning.com.sg](http://www.ace-learning.com.sg) – use of calculator to calculate trigo ratios)



### Instructions:

- Ensure your calculator is in DEG mode.
- To find the value of  $\sin 35^\circ$ , key in  $\sin 35 =$  You should obtain answer 0.574 (3sf)
- To find the value of an unknown angle eg.  $\theta$ , when you meet with a trigonometric ratio eg  $\sin \theta = 0.5$   
 $\rightarrow$  key in  $\text{2nd F } \sin^{-1} 0.5 =$   
 $\rightarrow$  You should obtain answer 30, which means  $\theta = 30^\circ$ .

### Practice 1:

Use your calculator to find the following, giving your answer correct to 4 sign. fig.

- $\sin 76^\circ = 0.97029 = 0.9703$  (4 sig. fig.)
- $\cos 125^\circ = -0.57357 = -0.5736$  (4 sig. fig.)
- $\tan 82^\circ = 7.1153 = 7.115$  (4 sig. fig.)
- $3 \tan 18.4^\circ + 5 \sin 69^\circ = 5.6658 = 5.666$  (4 sig. fig.)
- $\frac{\sin 43.6^\circ}{\tan 64.7^\circ + \cos 39.5^\circ} = 0.23885 = 0.2389$  (4 sig. fig.)

### Practice 2:

Find the value of the unknown angle  $\theta$  in each case, giving answers to 1 dp.

- $\sin \theta = 0.8 \rightarrow \theta = 53.1^\circ$
- $\cos \theta = 0.35 \rightarrow \theta = 69.5^\circ$
- $\tan \theta = 1.5 \rightarrow \theta = 56.3^\circ$

Now, we can learn how to find the sides of a right-angled triangle if an angle and one side is given.

**Example/Practice:**

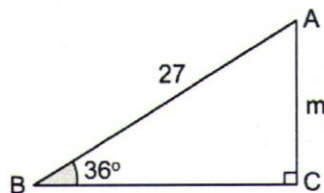
1. Find the value of  $m$ , giving your answer correct to 4 significant figures.

$$\sin 36^\circ = \frac{m}{27}$$

$$\sin 36^\circ = \frac{m}{27}$$

$$m = 15.870$$

$$m = 15.87 \quad (4 \text{ sig. fig.})$$

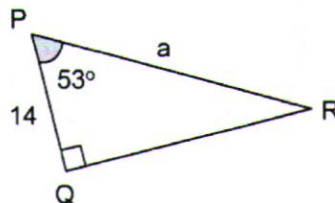


2. Calculate  $a$ , giving your answer correct to 4 significant figures.

$$\cos 53^\circ = \frac{14}{a}$$

$$a = 23.262$$

$$a = 23.26 \quad (4 \text{ sig. fig.})$$



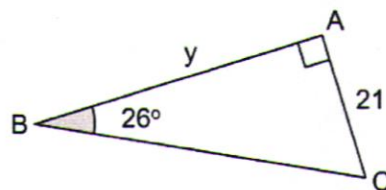
3. Find  $y$ , giving your answer correct to 4 significant figures.

$$\tan 26^\circ = \frac{21}{y}$$

$$\tan 26^\circ = \frac{21}{y}$$

$$y = 43.056$$

$$y = 43.06 \quad (4 \text{ sig. fig.})$$



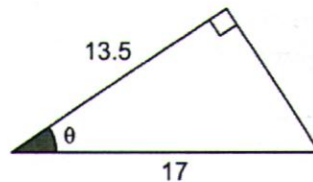
4. Find  $\theta$ , giving your answer correct to 1 decimal place.

$$\cos \theta = \frac{13.5}{17}$$

$$\cos \theta = \frac{13.5}{17}$$

$$\theta = 37.42$$

$$\theta = 37.4^\circ \quad (1 \text{ dec. pl.})$$



5. Find the values of  $x$  and  $y$ , giving your answers correct to 1 decimal place.

$$\sin x = \frac{24}{25}$$

$$\sin x = \frac{24}{25}$$

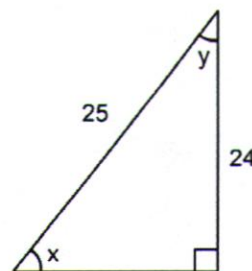
$$x = 73.73$$

$$x = 73.7^\circ \quad (1 \text{ dec. pl.})$$

$$\cos y = \frac{24}{25}$$

$$\cos y = \frac{24}{25}$$

$$y = 16.26 \quad y = 16.3^\circ \quad (1 \text{ dec. pl.})$$

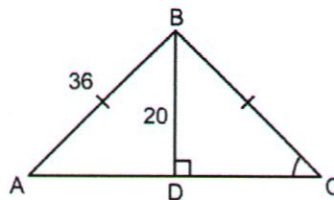


6. Given that triangle ABC is an isosceles triangle, find  $\hat{BCD}$  and give your answer correct to 1 decimal place.

$$\sin \hat{BCD} = \frac{20}{36}$$

$$\hat{BCD} = 33.74^\circ$$

$$\hat{BCD} = 33.7^\circ \text{ (1 dec. pl.)}$$



**Answers**

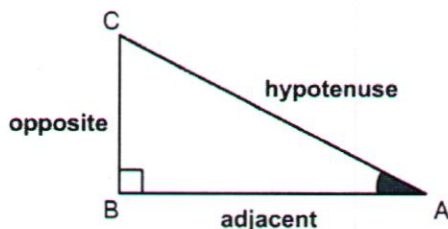
- 1)  $m = 15.87$     2)  $a = 23.26$     3)  $y = 43.06$     4)  $37.4^\circ$     5)  $x = 73.7^\circ, y = 16.3^\circ$     6)  $33.7^\circ$

**Summary**

Complete the diagram below.

**The 3 Basic Trigo Ratios**

The 3 basic Trigonometric Ratios are namely, **Sine Ratio**, **Cosine Ratio** and **Tangent Ratio**.



$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin A = \frac{b}{h}$$

$$\cos A = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos A = \frac{a}{h}$$

$$\tan A = \frac{\text{opposite}}{\text{adjacent}}$$

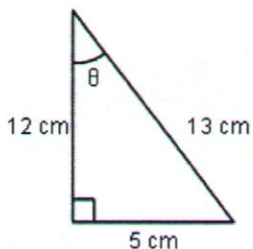
$$\tan A = \frac{b}{a}$$

**Homework**

Give all numerical answers to 3sf if it is inexact and all angles to 1 dpl.

1.

Write down the values of  $\sin \theta$ ,  $\cos \theta$  and  $\tan \theta$  as fractions in the table provided. Hence find the angle  $\theta$  in each case.



Trigonometric ratios	Fraction	$\theta$
$\sin \theta$	$\frac{5}{13}$	$22.6^\circ$
$\cos \theta$	$\frac{12}{13}$	$22.6^\circ$
$\tan \theta$	$\frac{5}{12}$	$22.6^\circ$

2. Write down the trigonometrical ratio for

a.  $\sin A$

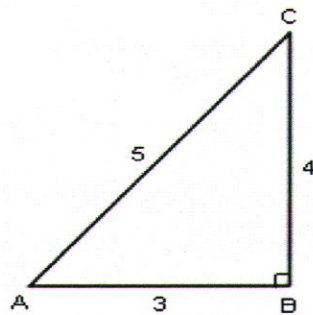
b.  $\cos C$

c.  $\tan A$

(a)  $\sin A = \frac{o}{h}$   
 $= \frac{4}{5}$

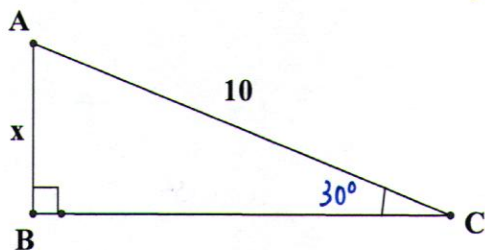
(b)  $\cos C = \frac{a}{h} = \frac{4}{5}$

(c)  $\tan A = \frac{4}{3}$



3. Find the value of  $x$  in each of the following diagrams.

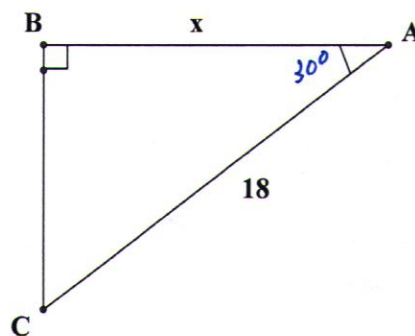
(a)



$\sin 30^\circ = \frac{x}{10}$

$x = 5$

(b)

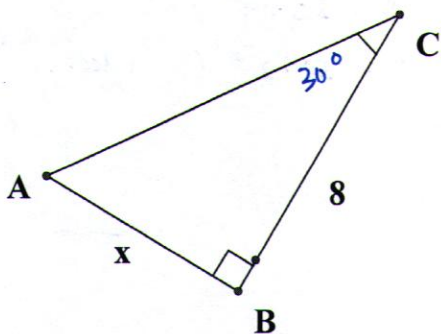


$\cos 30^\circ = \frac{x}{18}$

$x = 15.58$

$x = 15.6$  (3 sig. fig.)

(c)

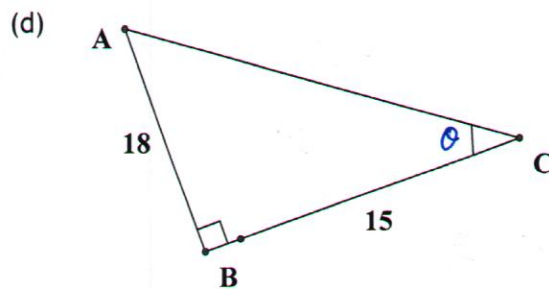
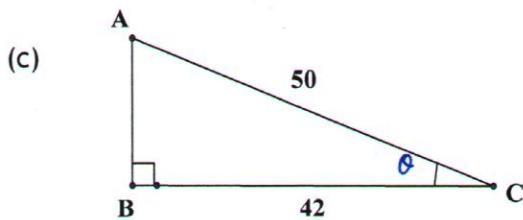
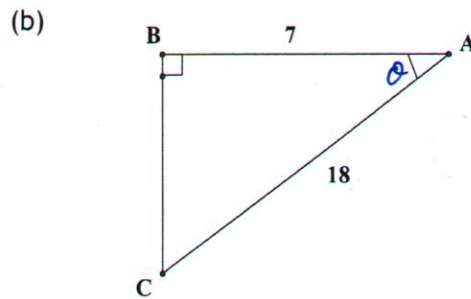
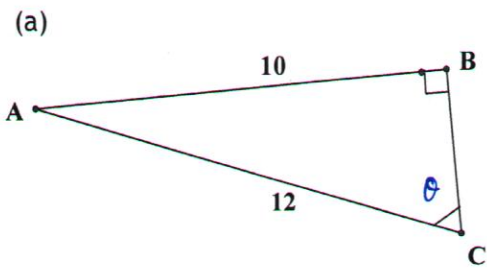


$\tan 30^\circ = \frac{x}{8}$

$x = 4.618$

$x = 4.62$  (3 sig. fig.)

4. Find the value of  $\theta$  in each of the following diagrams.



(a)  $\sin \theta = \frac{10}{12}$   
 $\theta = 56.44^\circ$   
 $\theta = 56.4^\circ$  (1 dec. pl.)

(b)  $\cos \theta = \frac{7}{18}$   
 $\theta = 67.11^\circ$   
 $\theta = 67.1^\circ$  (1 dec. pl.)

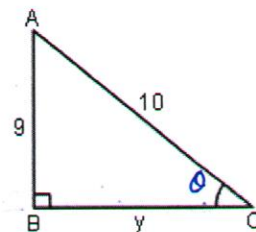
(c)  $\cos \theta = \frac{42}{50}$   
 $\theta = 32.85^\circ$   
 $\theta = 32.9^\circ$  (1 dec. pl.)

(d)  $\tan \theta = \frac{18}{15}$   
 $\theta = 50.19^\circ$   
 $\theta = 50.2^\circ$  (1 dec. pl.)

5. ABC is a right-angled triangle where AB = 9 cm, BC = y cm and AC = 10 cm.

a. Express  $\sin \theta$  as a fraction. Hence find the value of  $\theta$ .

$\sin \theta = \frac{9}{10}$   
 $\theta = 64.15^\circ$   
 $\theta = 64.2^\circ$  (1 dec. pl.)



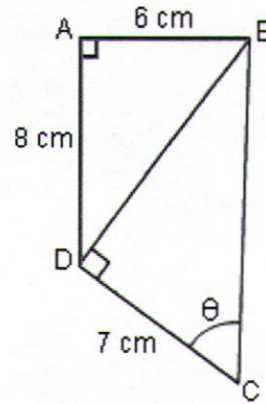
b. Hence or otherwise, find the length y.

$\cos 64.15^\circ = \frac{y}{10}$   
 $y = 4.36$  cm



6. The quadrilateral ABCD is made up of two right-angled triangles.

- a. Find the length BD.  
 b. Hence, find the angle  $\theta$ .



(a)  $BD = \sqrt{6^2 + 8^2}$

$BD = 10 \text{ cm}$

(b)  $\tan \theta = \frac{10}{7}$

$\theta = 55.00$

$\theta = 55.0^\circ$

**Answers:**

2a) 4/5

b) 4/5

c) 4/3

3a)  $x = 4.69$

b)  $x = 10.1$

c)  $x = 5.20$

4a)  $56.4^\circ$

b)  $67.1^\circ$

c)  $32.9^\circ$

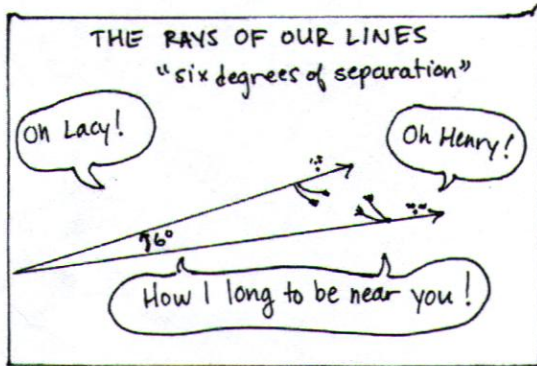
d)  $50.2^\circ$

5a)  $64.2^\circ$

b)  $y = 4.35$

6a)  $BD = 10 \text{ cm}$

b)  $55.0^\circ$



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**Deadline: Jun 2011**

My Reflection:

