West Spring Secondary School
2011 3E EM T1 Holiday Assignment

Name: ____________________________ (   )   **Class:** ___________   **Date:** __________

Overview

This worksheet covers the following:

1. **Indices & Standard Form**
2. **Quadratic Equations**
3. **Linear Inequalities**
4. **Congruence & Similarity**

Questions

1. Find the value of a in $\left(\sqrt{s^3\sqrt{s^{\frac{1}{2}}}}\right)^4 = s^a$, leaving your answer as a mixed number where necessary.

2. Simplify $\left(a^{-\frac{3}{4}}b^{\frac{1}{3}}\right)^2 \times \sqrt[3]{b} \div \sqrt{a^{-5}}$.

3. Given that $\sqrt{8.956} = 2.99$ and $\sqrt{89.56} = 9.46$, find the value of each of the following.

   (a) $\sqrt{0.8956}$

   (b) $\sqrt{89\,560}$

4. Evaluate

   (a) $\left(-\frac{1}{2}\right)^{-1} + \left(2\frac{3}{4} + 1\frac{1}{3}\right)^0$,

   (b) $-x(-x^x - x)^{-x}$ if $x = -2$,

   (c) $\dfrac{9^{\frac{1}{3}} \times 27^{-\frac{1}{2}}}{3^{-\frac{5}{6}}}$.

5. Simplify $\dfrac{(xy)^{-2}}{2z^0} \div \left(\dfrac{y}{x}\right)^{-2}$, giving your answer in positive index only.

6. Solve the equation, $6^{2x-1} \times 36^{x} = 1$.

7. Solve the equation $\dfrac{64}{4^{\frac{1}{2}}} \times 2^{-p} = 8^{p}$.

8. Solve the following equations.

(a) $9^{2c-5} + \dfrac{1}{9} = \dfrac{10}{81}$

(b) $\sqrt{\dfrac{a}{3}} = \dfrac{5}{a}$

9. A memory card has a storage capacity of 8 gigabytes. The average size of a picture file is 4.7 megabytes. Find the largest possible number of picture files that can be stored in the memory card, leaving your answer correct to the nearest whole number.

10. The population of the world was recently estimated to be 6,602,224,175 in March 2008 statistic report.

(a) Write 6,602,224,175 in standard form correct to 3 significant figures.

(b) The total land surface area of the world is approximately 1.5×10^{8} km^{2}. Using your answer in (a) and calculate the average number of people per square kilometre of land surface area. Leave your answer in standard form.

11. Solve the equation $t^2 - 5t - 20 = 0$ by completing the square method.

12. Rio was solving a quadratic equation using the 'completing the square' method but he could not get the desired solution. His first four steps of working are shown below. Identify the line that is wrong and continue solving for Rio.

Line 1: $2x^2 + 4x - 30 = 0$

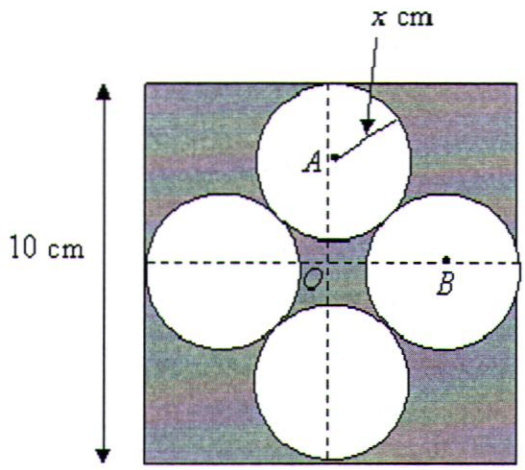
Line 2: $2x^2 + 4x = 30$

Line 3: $(2x+2)^2 - 4 = 30$

Line 4: $(2x+2)^2 = 30 + 4$

13. $x^2 + 6x - 5$ can be written as $(x + p)^2 + q$. Find p and q.

14.



The diagram shows a square of length 10 cm. O is the intersection of the diagonals of the square. Four smaller identical circles, touching each other, with radius x cm are drawn as shown. A and B are the centres of two of the smaller circles.

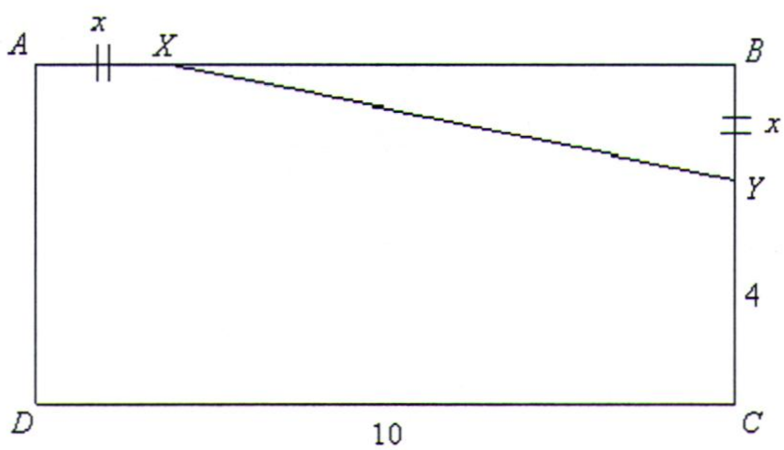
(a) Express the length of OA and AB in terms of x.

(b) Use Pythagoras Theorem to form an equation in x and show that it can be simplified to $x^2 + 10x - 25 = 0$.

(c) Solve the equation $x^2 + 10x - 25 = 0$, giving your answers correct to 2 decimal places.

(d) Calculate the area of the shaded region.

15. The diagram shows a rectangle $ABCD$. X and Y are points on AB and BC respectively such that $AX = BY$, $CD = 10$ cm, $CY = 4$ cm and the length of AX is x cm.



(a) Find in terms of x,

(i) BX,

(ii) the area of triangle BXY.

(b) Given that the area of the figure $ADCYX$ is 55 cm^2, form an equation in x and show that it reduces to $x^2 + 10x - 30 = 0$.

(c) Solve the equation $x^2 + 10x - 30 = 0$, giving both answers correct to two decimal places.

(d) Hence, find the area of triangle ABC.

16. Find the greatest integer value of x that satisfies the inequality $\frac{3-2x}{3} < \frac{3x+1}{7} < 5 - x$.

17. If $3k - 1 \leq 21 \leq 5k - 3$ write down

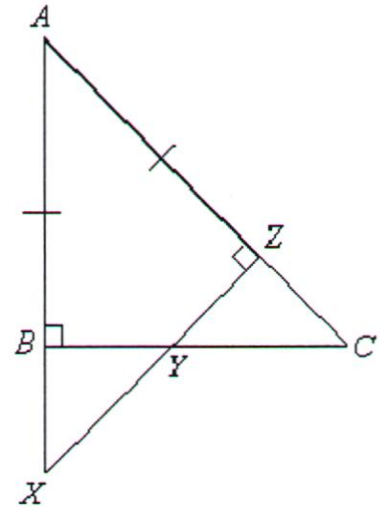
(a) the largest rational value of k,

(b) the smallest integer value of k,

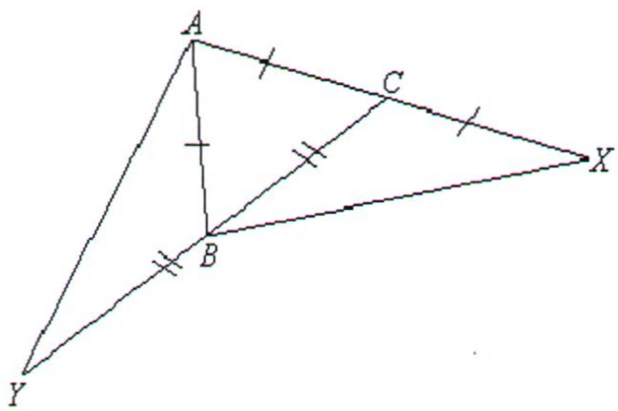
(c) the largest value of k if it is a prime number.

18. (a) In the diagram, $AZ = AB$. Stating your reasons clearly, show that ΔABC is congruent to ΔAZX.

(b) Given that $ZC = \frac{2}{5}AC$ and area of $\Delta CYZ = 10\ \text{cm}^2$, find the area of ΔAZX.



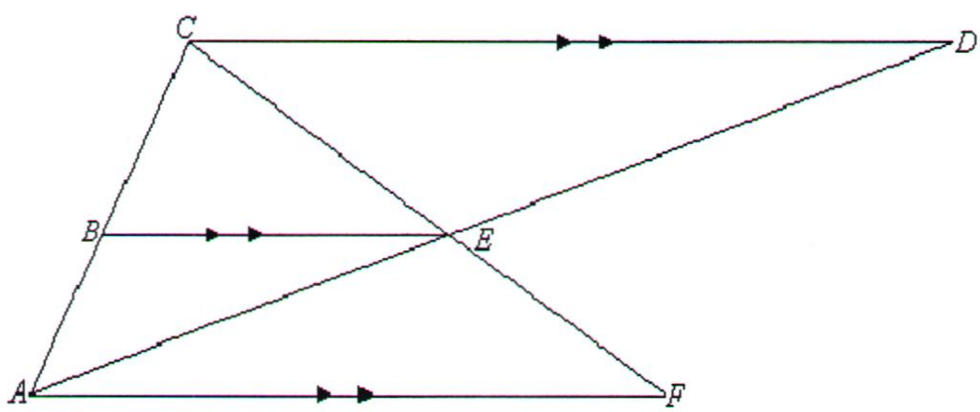
19.



(a) In the diagram, ACX and YBC are straight lines. It is given that $AB = AC = CX$ and $YB = BC$. Showing your working clearly, prove that two triangles are congruent.

(b) State the angle which has the same value as $\angle CBX$.

20. In the diagram, AF, BE and CD are parallel lines. AD and CF are straight lines and they meet at E.



(a) Prove that ΔAEF and ΔDEC are similar.

(b) It is given that $2CD = 3AF$. Find the ratio of the area of ΔCBE : area of ΔEAF.

2011 3Express EM T1 Holiday Assign. Solutions

1.

$$\left(\sqrt{s^3\sqrt{s^{\frac{1}{2}}}}\right)^4 = s^a$$

$$\left[\left(s^{3\frac{1}{4}}\right)^{\frac{1}{2}}\right]^4 = s^a$$

$$a = 6\frac{1}{2}$$

2.

$$\left(a^{-\frac{3}{4}}b^{\frac{1}{3}}\right)^2 \times \sqrt[3]{b} \div \sqrt{a^{-5}} = a^{-\frac{3}{2}}b^{\frac{2}{3}} \times b^{\frac{1}{3}} \div a^{-\frac{5}{2}}$$

$$= ab$$

3. (a)

$$\sqrt{0.8956} = 9.46 \times 10^{-1}$$
$$= 0.946$$

(b)

$$\sqrt{89560} = 2.99 \times 10^2$$
$$= 299$$

4. (a)

$$\left(2\frac{3}{4} + 1\frac{1}{3}\right)^0 = 1$$

$$\left(-\frac{1}{2}\right)^{-1} + \left(2\frac{3}{4} + 1\frac{1}{3}\right)^0 = -2 + 1$$
$$= -1$$

(b)

$$-(-2)(-(-2)^{-2} - (-2))^{-(-2)} = 6\frac{1}{8}$$

(c)

$$\frac{9^{\frac{1}{3}} \times 27^{-\frac{1}{2}}}{3^{-\frac{5}{6}}} = 3^{\frac{2}{3}} \times 3^{-\frac{3}{2}} \div 3^{-\frac{5}{6}}$$

$$= 3^0$$
$$= 1$$

5.

$$\frac{(xy)^{-2}}{2z^0} \div \left(\frac{y}{x}\right)^{-2} = \frac{(xy)^{-2}}{2} \div \left(\frac{y}{x}\right)^{-2}$$

$$= \frac{1}{2x^2y^2} \times \left(\frac{y^2}{x^2}\right)$$

$$= \frac{1}{x^4}$$

6.

$$6^{2x-1} \times 36^x = 1$$
$$6^{2x-1} \times 6^{2x} = 6^0$$

$$2x - 1 + 2x = 0$$

$$x = \frac{1}{4}$$

7.

$$\frac{64}{4^{\frac{1}{2}}}\times 2^{-p} = 8^{p}$$

$$\frac{2^{6}}{2}\times 2^{-p} = 2^{3p}$$

$$p = \frac{5}{4}$$

8. (a)

$$9^{2c-5} + \frac{1}{9} = \frac{10}{81}$$

$$9^{2c-5} = \frac{10}{81} - \frac{1}{9}$$

$$9^{2c-5} = \frac{1}{81}$$

$$9^{2c-5} = 9^{-2}$$

$$2c - 5 = -2$$

$$2c = 3$$

$$c = 1.5$$

8(b)

$$\sqrt{\frac{a}{3}} = \frac{5}{a}$$

$$\frac{a}{3} = \left(\frac{5}{a}\right)^{2}$$

$$\frac{a}{3} = \frac{25}{a^{2}}$$

$$a^{3} = 75$$

$$a = 75^{\frac{1}{3}}$$

$$a = 4.22$$

9.

$$4.7\times10^{6}\times x \le 8\times10^{9}$$

$$x \le 1702.13$$

Number of picture files stored = 1702

10. (a) 6.60×10^{9} (b) 4.4×10

(b)

$$\text{Number of people per square kilometer of land} = \frac{6.60\times10^{9}}{1.5\times10^{8}} = 4.4\times10$$

11.

$$t^{2} - 5t + \left(-\frac{5}{2}\right)^{2} = 20 + \left(-\frac{5}{2}\right)^{2}$$

$$\left(t - \frac{5}{2}\right)^{2} = \frac{105}{4}$$

$$t - \frac{5}{2} = \pm 5.1235$$

$$t = 7.62 \quad \text{or} \quad t = -2.62$$

12. Rio is wrong from Line 3 onwards, $x = 3$ or $x = -5$
[Solution]

$$2x^2+4x-30=0$$
$$2x^2+4x=30$$
$$x^2+2x=15$$
$$(x+1)^2-1=15$$
$$(x+1)^2=16$$
$$x+1=4 \quad \text{or} \quad x+1=-4$$
$$x=3 \quad \text{or} \quad x=-5$$

13.

$$x^2+6x-5=x^2+2(3)x+9-9-5$$
$$=(x+3)^2-14$$
$$\therefore p=3,\ q=-14$$

14.a) $OA=5-x, AB=2x$ (c) $x=2.07$ (d) 46.1 cm^2

(b)

$$(5-x)^2+(5-x)^2=(2x)^2$$
$$2(25-10x+x^2)=4x^2$$
$$x^2+10x-25=0$$

(c)

$$x=\frac{-10\pm\sqrt{200}}{2}$$
$$=2.07$$

(d) $10\times10-4\times(3.142\times2.07\times2.07)=46.1\ \text{cm}^2$

15) (a) (i) $10-x$ (ii) $\frac{1}{2}(10-x)(x)$ (c) $x=2.42$ or -12.42 (d) 32.1 cm^2

[Solution]

(b)

$$10(4+x)-\frac{1}{2}(10-x)(x)=55$$
$$80+20x-10x+x^2=110$$
$$x^2+10x-30=0$$

(c)

$$x=\frac{-(10)\pm\sqrt{10^2-4(1)(-30)}}{2(1)}$$
$$=\frac{-(10)\pm\sqrt{220}}{2}$$
$$\approx 2.4161 \quad \text{or} \quad \approx -12.4161$$
$$\approx 2.42 \quad \text{or} \quad \approx -12.42 \ \text{(rejected)}$$

(d)

$$\text{Area}=\frac{1}{2}(10)(4+2.4161)$$
$$=32.1\ \text{cm}^2$$

16) $x=3$

[Solution]

$$21-14x<9x+3 \quad \text{and} \quad 3x+1<35-7x$$
$$18<23x \qquad 10x<34$$
$$\frac{18}{23}<x \qquad x<\frac{34}{10}$$

17)

(a) $7\frac{1}{3}$ (b) 5 (c) 7

[Solution]

(a)

$3k-1 \le 21$ and $21 \le 5k-3$

$3k \le 22$ $24 \le 5k$

$k \le 7\frac{1}{3}$ $k \ge 4\frac{4}{5}$

$4\frac{4}{5} \le k \le 7\frac{1}{3}$

Largest rational number = $7\frac{1}{3}$

18. (a)

$AZ = AB$

$\angle AZX = \angle ABC$

$= 90^\circ$

$\angle ZAZ = \angle CAB$ (Common)

$\Delta ABC = \Delta AZX (AAS)$

(b) 40 cm^2

19. (a) $\Delta ABY \equiv \Delta XCB$ (SAS) (b) $\angle BYA$

[Solution]

(a) $AB = CX$ (given)

$BY = CB$ (given)

$\angle ABY = \angle XCB$ ($\angle$ on a strt line)

20 (b) $\frac{9}{10}$

[Solution]

(a) $\angle AFE = \angle DCE$ (alt. $\angle$s)

$\angle AEF = \angle DEC$ (vert. opp. $\angle$s)