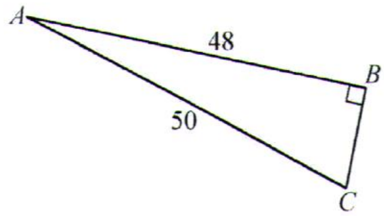
Topic: Pythagoras' Theorem

Name: ________________________ (   )   Date:____________

Class: ___________

1. In ΔABC, $\angle B = 90°$, $AB = 48$ cm and $AC = 50$ cm.



(a) Find the length of BC.
(b) Hence, find the area of ΔABC.

Solution:

(a) $\angle B = 90°$ (given)

$AC^2 = AB^2 + BC^2$ (Pythagoras' Theorem)

$50^2 = 48^2 + BC^2$

$BC^2 = 50^2 - 48^2$

$= 196$

$\therefore BC = \sqrt{196}$

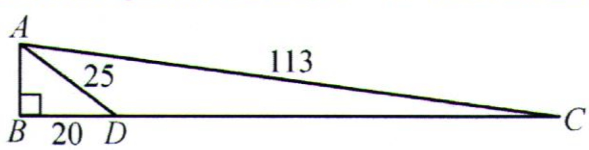
$= 14\,\text{cm}$

(b) Area of $\Delta ABC = \frac{1}{2} \times AB \times BC$

$= \frac{1}{2} \times 48 \times 14$

$= 336 \text{ cm}^2$

2. In the diagram, BDC is a straight line and $\angle ABC = 90°$. The dimensions given are in m.



Find the length of
(a) AB,
(b) DC.

Solution:

(a) In ΔABD,

$\angle ABD = 90^\circ$

$AD^2 = AB^2 + BD^2$ (Pythagoras' Theorem)

$25^2 = AB^2 + 20^2$

$AB^2 = 25^2 - 20^2$

$= 225$

$\therefore AB = \sqrt{225}$

$= 15\text{ m}$

(b) In ΔABC,

$AC^2 = AB^2 + BC^2$ (Pythagoras' Theorem)

$113^2 = 15^2 + BC^2$

$BC^2 = 113^2 - 15^2$

$= 12\,544$

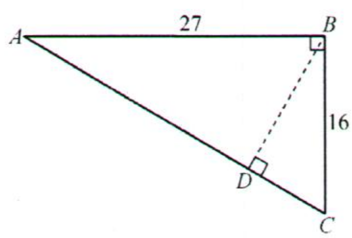
$BC = \sqrt{12\,544}$

$= 112\text{ m}$

$\therefore DC = 112 - 20$

$= 92\text{ m}$

3. In the diagram, ADC is a straight line and $\angle ABC = 90^\circ$. The line BD is perpendicular to AC. The dimensions given are in cm.



(a) Find the area of ΔABC.
(b) Find the length of AC.
(c) Hence or otherwise, calculate the length of BD.

Solution:

(a) Area of $\Delta ABC = \frac{1}{2} \times AB \times BC$

$= \frac{1}{2} \times 27 \times 16$

$= 216\text{ cm}^2$

(b) In ΔABC,

$$AC^2 = AB^2 + BC^2 \text{ (Pythagoras' Theorem)}$$

$$AC^2 = 27^2 + 16^2$$

$$= 985$$

$$AC = \sqrt{985}$$

$$= 31.4 \text{ cm (correct to 3 sig. fig.)}$$

(c) Area of $\Delta ABC = 216 \text{ cm}^2$

$$\therefore \frac{1}{2} \times AC \times BD = 216$$

$$\frac{1}{2} \times \sqrt{985} \times BD = 216$$

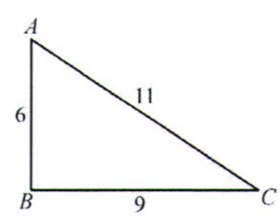
$$BD = \frac{432}{\sqrt{985}}$$

$$= 13.8 \text{ cm (correct to 3 sig. fig.)}$$

4. Determine whether each of the following triangles is a right-angled triangle.
   (a) ΔABC with $AB = 6$ cm, $BC = 9$ cm and $CA = 11$ cm.
   (b) ΔXYZ with $XY = 9$ cm, $YZ = 40$ cm and $ZX = 41$ cm.
   (c) ΔPQR with $PQ = 15$ m, $QR = 112$ m and $RQ = 113$ m.

Solution:

(a)



$$AC^2 = 11^2$$

$$= 121$$

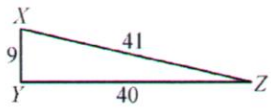
$$AB^2 + BC^2 = 6^2 + 9^2$$

$$= 117$$

$$\therefore AC^2 \neq AB^2 + BC^2$$

Hence, ΔABC is not a right-angled triangle.

(b)



$$XZ^2 = 41^2$$
$$= 1681$$

$$XY^2 + YZ^2 = 9^2 + 40^2$$
$$= 1681$$

$$\therefore XZ^2 = XY^2 + YZ^2$$

Hence, ΔXYZ is a right-angled triangle.
(converse of Pythagoras' Theorem)

(c)

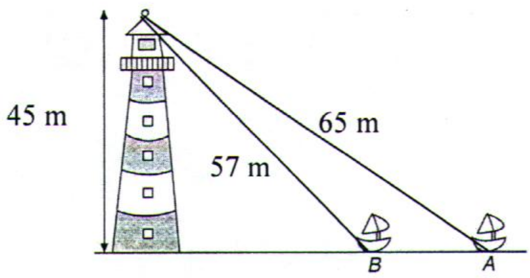
$$RQ^2 = 113^2$$
$$= 12\ 769$$

$$PQ^2 + PR^2 = 15^2 + 112^2$$
$$= 12\ 769$$

$$\therefore RQ^2 = PQ^2 + PR^2$$

Hence, ΔPQR is a right-angled triangle.
(converse of Pythagoras' Theorem)

5.



From the top of a lighthouse, 45 m above the sea, two boats are spotted.

(a) Show that the distance between Boat A and Boat B is approximately 11.9 m.

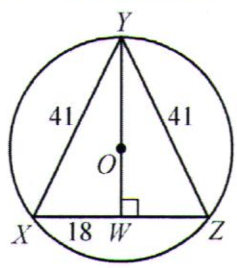
(b) Calculate the time taken by Boat A to reach Boat B if the speed of Boat A is 5 m/s.

Solution:

[Answer Key]

(b) 2.38 s

6. In the diagram, an isosceles triangle XYZ is inscribed in a circle with centre O. The line WY is perpendicular to the line XZ, $XY = ZY = 41$ cm and $XZ = 18$ cm.



(a) Find the length of WY.

(b) Find the radius of the circle.

(c) Hence, find

(i) the length of OW,

(ii) the area of ΔXOY.

Solution:

(a) In ΔWXY,

$$XY^2 = XW^2 + WY^2 \text{ (Pythagoras' Theorem)}$$

$$41^2 = 9^2 + WY^2$$

$$WY^2 = 41^2 - 9^2$$

$$= 1600$$

$WY = \sqrt{1600}$

$= 40$ cm

(a) Let the radius of the circle be r cm.

$OW = (40 - r)$ cm

In ΔXOW,

$OX^2 = OW^2 + XW^2$ (Pythagoras' Theorem)

$r^2 = (40 - r)^2 + 9^2$

$r^2 = 1600 - 80r + r^2 + 81$

$80r = 1681$

$r = 21.0$ cm (correct to 3 sig. fig.)

$\therefore$ radius of the circle is 21.0 cm.

(b) (i) $OW = 40 - \dfrac{1681}{80}$

$= 19.0$ cm (correct to 3 sig. fig.)

(ii) Area of ΔXOZ $= \dfrac{1}{2} \times 18 \times \left(40 - \dfrac{1681}{80}\right)$

$= 170.9\ \text{cm}^2$ (correct to 4 sig. fig.)

Area of ΔXYZ $= \dfrac{1}{2} \times 18 \times 40$

$= 360\ \text{cm}^2$

Area of ΔXOY $= \left(\dfrac{360 - 170.9}{2}\right)$

$= 94.6\ \text{cm}^2$ (correct to 3 sig. fig.)

OR

Area of ΔXOY = area of ΔXWY – area of ΔXWO

$= \dfrac{1}{2} \times 9 \times 40 - \dfrac{1}{2} \times 9 \times \left(40 - \dfrac{1681}{80}\right)$

$= 94.6\ \text{cm}^2$

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