West Spring Secondary School
3E Elementary Mathematics Assignment 4: Properties of Circles

Name: _____ ( ) Class: _____ Date: _____

Recap & Summary

1. Symmetrical Properties of Circles
   a. Perpendicular bisecting chord ✓
   b. Equal chords are equidistant from centre ✓
2. Angle Properties of Circles:
   a. ∠ at centre = twice at circumference ✓
   b. ∠ in a semicircle ✓
   c. ∠ in the same segment ✓
3. Angle Properties of Cyclic Quadrilaterals:
   a. Opposite ∠s of cyclic quadrilaterals ✓
   b. Exterior ∠s of cyclic quadrilaterals ✓
4. Tangents & their properties:
   a. Tangent ⊥ radius ✓
   b. Tangents from an external point ✓
   c. Alternate Segment Theorem ✓

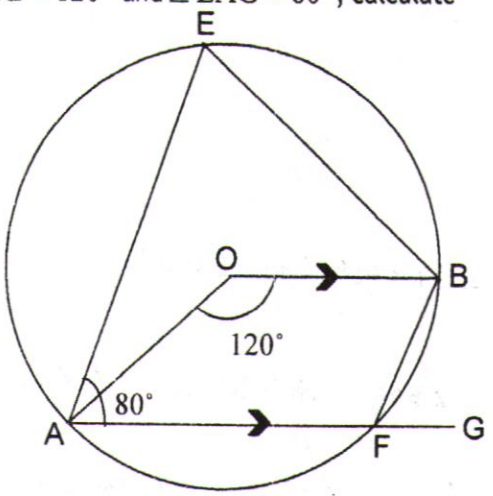
Overview

This worksheet is a revision of all the above.

Practice

Show all workings with their suitable reasoning.

1. In the diagram, O is the centre. A, B, E and F are points on the circumference of the circle. AFG is a straight line and AG is parallel to OB. Given that $\angle AOB = 120^\circ$ and $\angle EAG = 80^\circ$, calculate
   a. $\angle AEB$
   b. $\angle BFG$
   c. $\angle EBO$



(a) $\angle AEB = 120^\circ \div 2$ (∠ at centre = 2 ∠ at circumference)
$= 60^\circ$ //

(b) $\angle AFB = 180^\circ - 60^\circ$ (opp. ∠s of a cyclic quad.)
$= 120^\circ$

$\angle BFG = 180^\circ - 120^\circ$ (adj. ∠s on a str. line)
$= 60^\circ$

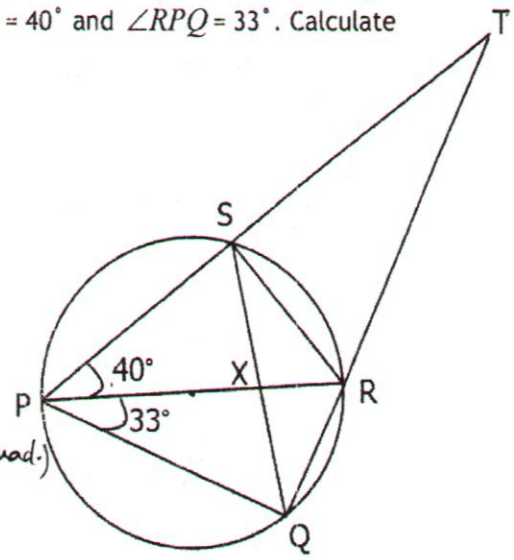
OR

$\angle BFG = 60^\circ$ (exterior ∠ of cyclic quad.) //

c) $\angle EBF = 180^\circ - 80^\circ$ (opp. ∠s of a cyclic quad.)
$= 100^\circ$
$\angle OBF = 60^\circ$ (alt. ∠s)
$\angle EBO = 100^\circ - 60^\circ$
$= 40^\circ$ //

2. In the diagram, PR is the diameter of the circle $PQRS$. The straight lines PST and QRT cut the circle at P, Q, R and S. PR and QS cut at X. $\angle SPR = 40^\circ$ and $\angle RPQ = 33^\circ$. Calculate

a. $\angle PRS$
b. $\angle QRS$
c. $\angle PSX$
d. $\angle PTQ$



a) $\hat{PSR} = 90^\circ$ (rt.$\angle$ in a semicircle)

$\hat{PRS} = 180^\circ - 90^\circ - 40^\circ$ ($\angle$ sum of $\triangle$)

$= 50^\circ$

b) $\angle QRS = 180^\circ - 40^\circ - 33^\circ$ (opp. $\angle$s of a cyclic quad.)

$= 107^\circ$

c) $\angle QSR = 33^\circ$ ($\angle$s in the same segment)

$\angle PSX = 90^\circ - 33^\circ$

$= 57^\circ$

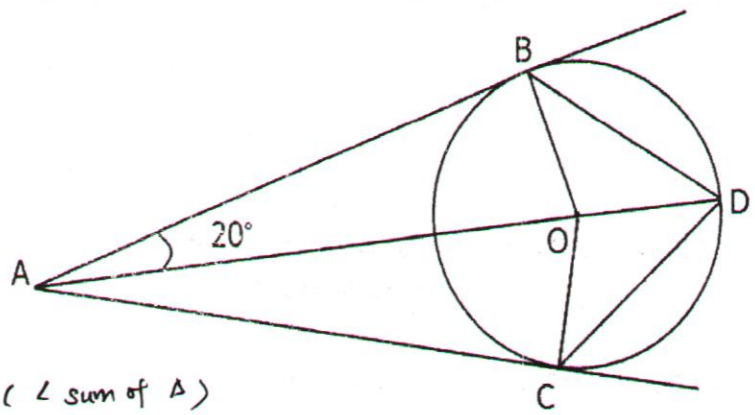
d) $\angle PQR = 90^\circ$ (rt. $\angle$ in a semicircle)

$\angle PTQ = 180^\circ - 90^\circ - 73^\circ$ ($\angle$ sum of $\triangle$)

$= 17^\circ$

3. In the diagram, AB and AC are tangents to the circle at B and C respectively and O is the centre of the circle. Given that $\angle OA1$

a. $\angle OBA$,
b. $\angle BOC$,
c. $\angle BDO$
d. $\angle BOD$



(a) $\angle OBA = 90^\circ$ (tan $\perp$ rad)

(b) $\angle BOA = 180^\circ - 90^\circ - 20^\circ$ ($\angle$ sum of $\triangle$)

$= 70^\circ$

$\angle BOC = 70^\circ \times 2$ (tangents from an external pt.)

$= 140^\circ$

d) $\hat{BOD} = 180^\circ - 70^\circ$ (adj. $\angle$s on a str. line)

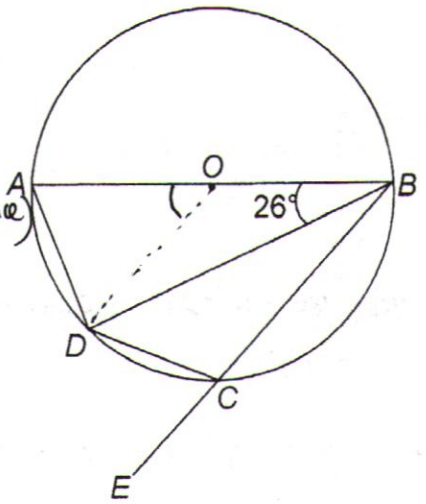
$= 110^\circ$

c) $\angle BDO = (180^\circ - 110^\circ) \div 2$ (base $\angle$s of isos. $\triangle$)

$= 35^\circ$

4. In the diagram, *AB* is a diameter of the circle with centre *O*, and *BCE* is a straight line. $A\hat{B}D = 26°$. Showing all the reasons, find

a. $\angle AOD$
b. $\angle ADB$
c. $\angle BAD$
d. $\angle BCD$



(a) $\angle AOD = 26° \times 2$ ($\angle$ at centre = 2 $\angle$ at circumference)
$= 52°$

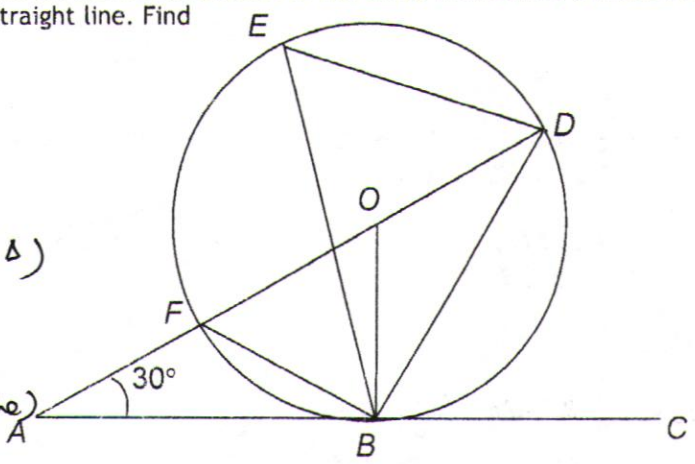
(b) $\angle ADB = 90°$ (rt. $\angle$ in a semicircle)

(c) $\angle BAD = 180° - 90° - 26°$ ($\angle$ sum of $\triangle$)
$= 64°$

(d) $\angle BCD = 180° - 64°$ (opp. $\angle$s in a cyclic quad.)
$= 116°$

5. In the diagram, *B*, *D*, *E* and *F* are points on the circumference of the circle with centre *O*. ABC is a tangent to the circle at *B*. AFD is a straight line. Find

a. $\angle AOB$
b. $\angle DEB$
c. $\angle DBC$



(a) $\angle ABO = 90°$ (tan $\perp$ rad.)

$\angle AOB = 180° - 30° - 90°$ ($\angle$ sum of $\triangle$)
$= 60°$

(b) $\angle BOD = 180° - 60°$ (adj. $\angle$s on a str. line)
$= 120°$

$\angle DEB = 120° \div 2$ ($\angle$ at centre = 2 $\angle$ at circumference)
$= 60°$

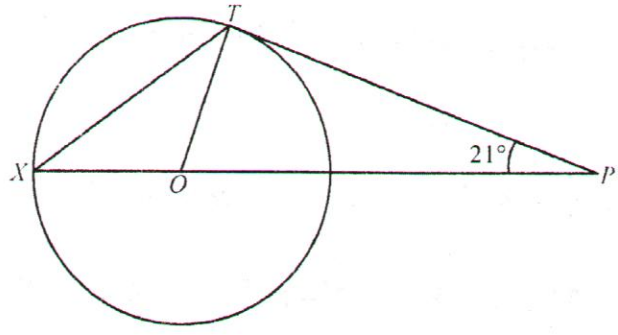
(c) $\angle DBC = \angle DEB$ ($\angle$s in the alt. segment)
$= 60°$

Answers:

1a) 60°	1b) 60°	1c) 40°	2a) 50°	2b) 107°	2c) 57°	2d) 17°
3a) 90°	3b) 140°	3c) 35°	3d) 110°	4a) 52°	4b) 90°	4c) 64° 4d) 116°
5a) 60°	5b) 60°	5c) 60°				

Homework

1. In the diagram, PT is a tangent to the circle with centre O and XOP is a straight line. Given that $\angle TPO = 21°$, find
   a) $\angle TOP$,
   b) $\angle OXT$.

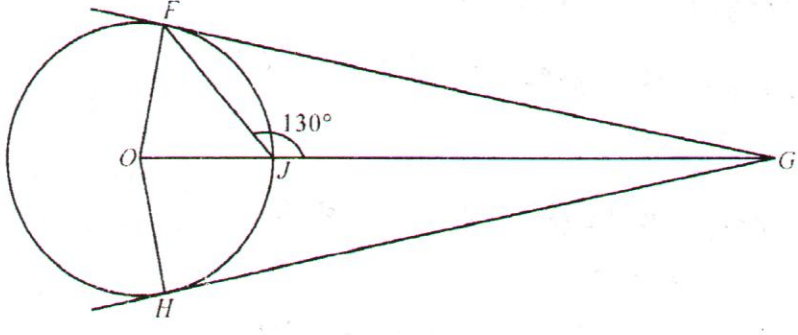


(a) $\hat{TOP} = 180° - 90° - 21°$ ($\angle$ sum of Δ)
$= 69°$

(b) $\hat{XOT} = 180° - 69°$ (adj. $\angle$s on a str. line)
$= 111°$

$\hat{OXT} = (180° - 111°) \div 2$ (base $\angle$s of isos. Δ)
$= 34.5°$

2. In the diagram, FG and HG are tangents to the circle with centre O and OJG is a straight line. If $\angle FJG = 130°$, calculate
   a) $\angle JFG$,
   b) $\angle FGH$.



(a) $\angle JFO = 180° - 130°$ (adj. $\angle$s on a str. line)
$= 50°$

$\angle JFG = 90° - 50°$
$= 40°$ //

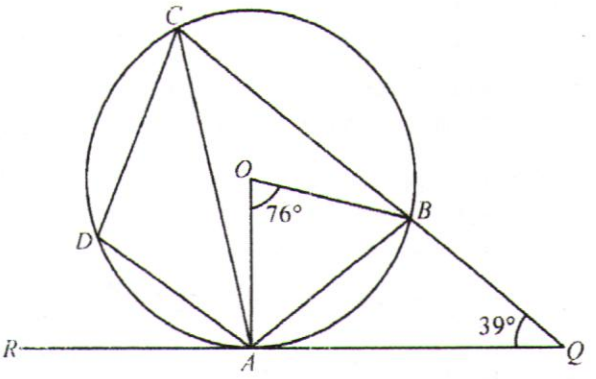
(b) $\angle FGO = 180° - 130° - 40°$ ($\angle$ sum of Δ)
$= 10°$

$\angle FGH = 10° \times 2$ (tangents from an external pt.)
$= 20°$ //

3. In the diagram, O is the centre of the circle and RAQ is a tangent to the circle at A. Given that $\angle AOB = 76°$ and $\angle AQB = 39°$, calculate

a) $\angle BAQ$,

b) $\angle ADC$.



(a) $B\hat{A}O = (180° - 76°) \div 2$ (base $\angle$s of a isos. $\triangle$)

$= 52°$

$B\hat{A}Q = 90° - 52°$

$= 38°$

(b) $A\hat{B}C = 38° + 39°$ ($\angle$ ext. $\angle$ = opp. int. $\angle$s)

$= 77°$

$A\hat{D}C = 180° - 77°$ (opp. $\angle$s in a cyclic quad.)

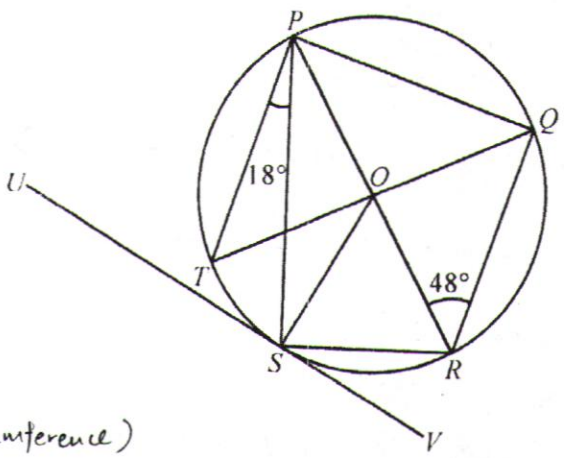
$= 103°$

4. In the diagram, a circle with centre O passes through points P, Q, R, S and T and UV is a tangent to the circle. Given that $\angle PRQ = 29°$, $\angle TPS = 18°$ and TOQ is a straight line, calculate

a) $\angle QPR$,

b) $\angle SOR$, (Hint: $\angle SOT = 2\angle SPT$)

c) $\angle RSV$.



(a) $\angle QPR = 180° - 90° - 48°$ ($\angle$ sum of $\triangle$)

$= 42°$

(b) $O\hat{P}T = 48°$ ($\angle$s in the same segment)

$O\hat{P}S = 48° - 18°$

$= 30°$

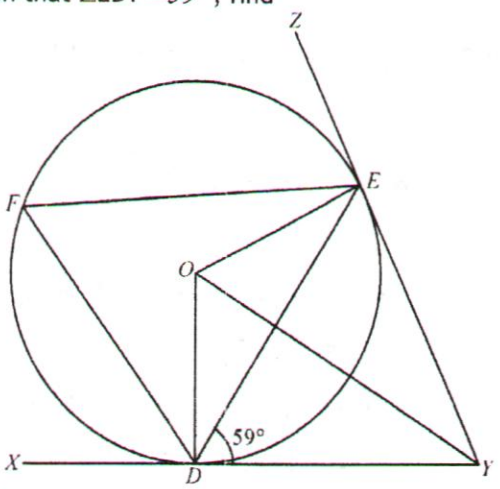
$S\hat{O}R = 60°$ ($\angle$ at centre = 2 $\angle$ at circumference)

(c) $R\hat{S}V = S\hat{P}R$ ($\angle$s in the alt. segment)

$= 30°$

5. In the figure, O is the centre of the circle passing through points D, E and F. XY and ZY are tangents to the circle at points D and E respectively. Given that $\angle EDY = 59°$, find

a) $\angle DFE$,
b) $\angle DOY$,
c) $\angle DYE$.



(a) $\angle DFE = 59°$ ($\angle$s in the alt. segment)

(b) $\angle DOE = 59° \times 2$ ($\angle$ at centre = 2$\angle$ at circumference)
$= 118°$

$\angle DOY = 118° \div 2$ (tangents from an external pt.)

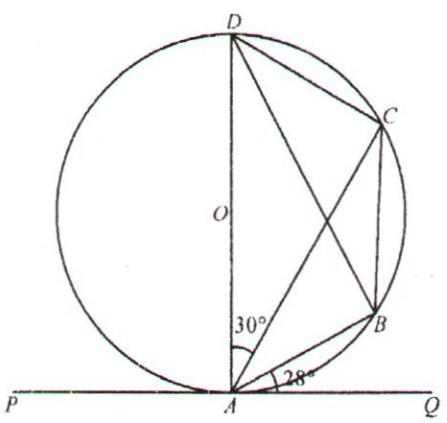
(c) $= 59°$

$\angle DYO = 180° - 90° - 59°$ ($\angle$ sum of $\triangle$)
$= 31°$

$\angle DYE = 31° \times 2$ (tangents from an external pt.)
$= 62°$

6. In the diagram, PAQ is the tangent to the circle at A. AD is a diameter of the circle. Given that $\angle DAC = 30°$ and $\angle BAQ = 28°$, find

a) $\angle ABD$,
b) $\angle CBD$,
c) $\angle ABC$,
d) $\angle BCD$.



(a) $\angle ABD = 90°$ (rt. $\angle$ in a semicircle)

(b) $\angle CBD = 30°$ ($\angle$s in the same segment)

(c) $\angle ABC = 90° + 30°$
$= 120°$

(d) $\angle ACB = 28°$ ($\angle$s in the alt. segment)

$\angle BCD = 90° + 28°$
$= 118°$

Answers:

1a) 69°	1b) 34.5°	2a) 40°	2b) 20°	3a) 38°	3b) 103°
4a) 42°	4b) 60°	4c) 30°	5a) 59°	5b) 59°	5c) 62°
6a) 90°	6b) 30°	6c) 120°	6d) 118°		