West Spring Secondary School
2011 3E EM Assignment 29: Properties of Circles

Name: ______________________ ( ) Class: __________ Date: _________

Overview

This worksheet covers the following:

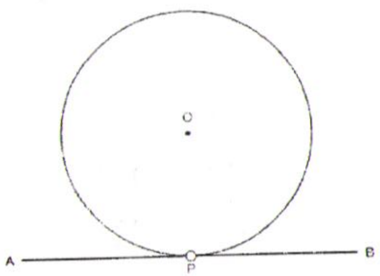
1. Tangents & their properties:
   a. Tangent ⊥ radius
   b. Tangents from an external point
   c. Alternate Segment Theorem

Introduction

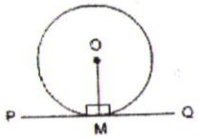
What is a tangent?

Definition

A straight line touching the circle at one single point is called a **tangent** to the circle And that point is called the **point of contact**



Property 1: Tangent ⊥ radius



A tangent to a circle is perpendicular to the radius drawn to the point of contact.

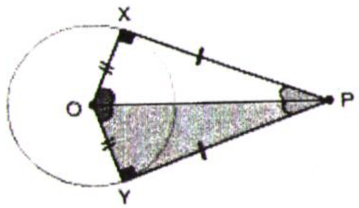
This property can be abbreviated as tan ⊥ rad.

Property 2: Tangents from an External point

P is a point outside the circle, with centre O, PX and PY are two tangents drawn from P To touch the circle at X and Y respectively.

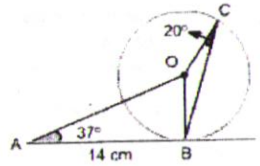
The following are true:

a. Two tangents to a circle from an external point are equal in length.
b. The line joining the external point to the centre of the circle bisects:
   a. the angle between the tangents,
   b. the angle between the radii.



Practice

1. AB is a tangent to the circle, with centre O. Given that AB = 14 cm, $O\hat{A}B = 37°$ and $O\hat{C}B = 20°$.
   Find (i) $A\hat{O}B$, (ii) $B\hat{O}C$ (Ans: 53°, 140°)

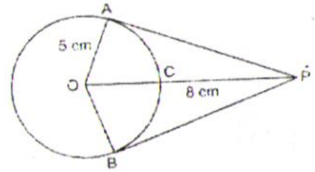


(i) $A\hat{B}O = 90°$ (tan ⊥ rad.)

$A\hat{O}B = 180° - 90° - 37°$ (∠ sum of △)
$= 53°$

(ii) $B\hat{O}C = 180° - 20° - 20°$ (base ∠s of isos. △)
$= 140°$

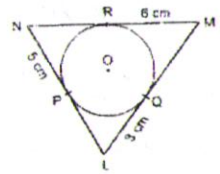
2. Calculate AP and BP given OA = 5cm and CP = 8 cm.
(Ans: AP = 12cm, BP=12cm)



$AP = \sqrt{13^2 - 5^2}$
$= 12$ cm

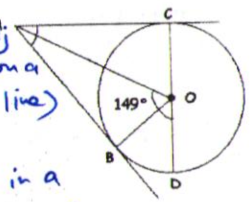
$AP = BP$ (tangents from an external point)
$= 12$ cm

3. LM, LN and MN are tangents to the circle with centre O. Find the perimeter of ΔLMN.
(Ans: 28 cm)



Perimeter $= (5+3+6) \times 2$
$= 28$ cm

4. **In the diagram, AB and AC are tangents to the circle at B and C respectively. O is the centre of the circle and $A\hat{O}D = 149°$. What is the angle, $B\hat{A}C$?** (Ans: 118°)



$A\hat{O}C = 180° - 149°$ (adj ∠s on a str. line)
$= 31°$

$A\hat{C}O = 90°$ (rt. ∠ in a semicircle)

$C\hat{A}O = 180° - 90° - 31°$ (∠ sum of △)
$= 59°$

$B\hat{A}C = 59° \times 2$ (tangents from an external pt.)
$= 118°$

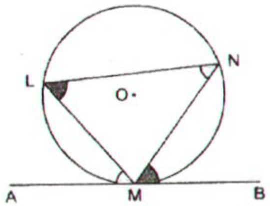
Property 3: Alternate Segment Theorem

> **Alternate Segment Theorem**
>
> The **angle between the tangent and a chord** through the **point of contact** is equal to the angle in the alternate theorem.

$\angle BMN = \angle MLN$

$\angle LMA = \angle LNM$

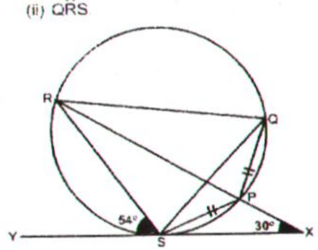
This property can be abbreviated as: (∠s in alt. segment)



Practice

1.

The points P, Q, R and S lie on a circle where XY is a tangent to the circle at S. $R\hat{S}Y = 54°$, $R\hat{X}Y = 30°$, PQ = PS and XPR is a straight line. Calculate (i) $P\hat{S}X$ (ii) $Q\hat{R}S$



(i) $S\hat{P}R = 54°$ (∵∠s in alt. segment)

$S\hat{P}X = 180° - 54°$ (adj. ∠s on a str. line)

$= 126°$

$P\hat{S}X = 180° - 126° - 30°$ (∠ sum of Δ)

$= 24°$

(ii) $S\hat{Q}P = P\hat{S}X$ (∠s in the alt. segment)

$= 24°$

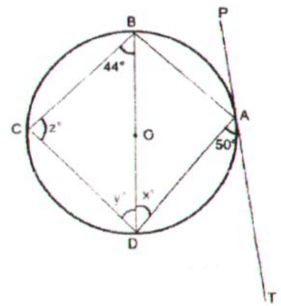
$S\hat{P}Q = 180° - 24° - 24°$ (∠ sum of Δ)

$= 132°$

$Q\hat{R}S = 180° - 132°$ (opp. ∠s in a cyclic quad.)

$= 48°$

2. **Let PAT be the tangent to the circle with centre O. Find the values of x , y and z.**



$\angle z = 90°$ (rt. ∠ in a semicircle)

$\angle y = 180° - 90° - 44°$ (∠ sum of Δ)

$= 46°$

$\angle BAP = 180° - 90° - 50° = 40°$ (adj. ∠s on a str line)

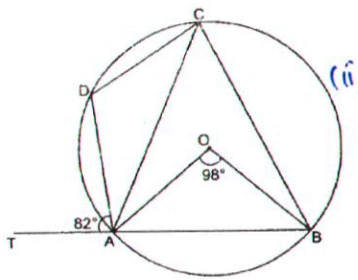
$\angle x = \angle BAP$

$= 40°$ (∠s in the alt. segment)

3. **In the diagram, O is the centre of the circle, TAB is a straight line, ∠AOB = 98° and ∠TAD = 82°**

Find the value of i) ∠ACB

ii) ∠ACD



(i) $\angle ACB = 98° \div 2$ (∠ at centre = 2∠ at circumference)

$= 49°$

(ii) $B\hat{C}D = D\hat{A}T$ (ext. ∠ of a cyclic quad.)

$= 82°$

$A\hat{C}D = 82° - 49°$

$= 33°$

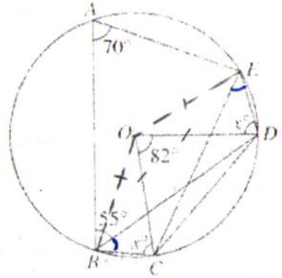
Answers:

1i) 24° 1ii) 48° 2) x=40°, y=46°, z=90° 3i) 48° 3ii) 33°

Homework

1. O is the centre of the circle through A, B, C, D, and E.
$\angle COD = 82°$, $\angle ABD = 55°$, $\angle BAE = 70°$ and $\angle BCO = \angle EDO = x°$.

Calculate
(a) $\angle ODC$,
(b) $\angle CED$,
(c) $\angle AEC$,
(d) x.



(a) $\angle ODC = (180° - 82°) \div 2$ (base $\angle$s of isos Δ)
$= 49°$

(b) $\angle CED = 82° \div 2$ ($\angle$ at centre $= 2\angle$ at circumference)
$= 41°$

(c) $\angle CBD = \angle CED$ ($\angle$s in the same segment)
$= 41°$

$\angle AEC = 180° - 55° - 41°$ (opp. $\angle$s in a cyclic quad.)
$= 84°$

$B\hat{O}E = 70° \times 2$ ($\angle$ at centre $= 2\angle$ at circumference)
$= 140°$

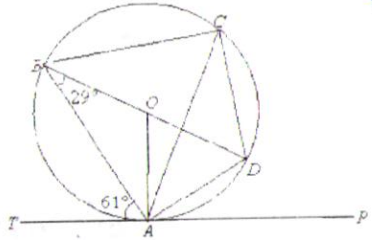
$B\hat{O}C = (140° - 82°) \div 2$
$= 29°$

$x = (180° - 29°) \div 2$ ($\angle$ at centre $= 2\angle$ at circumference)
$= 75.5°$

2. In the diagram, TAP is a tangent to the circle $ABCD$ at A. $T\hat{A}B = 61°$ and $A\hat{B}D = 29°$. O is the centre of circle $ABCD$.

(a) Find, stating your reasons clearly,
(i) $\angle AOD$
(ii) $\angle ACD$
(iii) $\angle BCD$
(iv) $\angle BCA$
(v) $\angle PAD$

(b) What is the special name given to quadrilateral $ABCD$?



(i) $\angle AOD = 29° \times 2$ ($\angle$ at centre $= 2\angle$ at circumference)
$= 58°$

(ii) $\angle ACD = 29°$ ($\angle$s in the same segment)

(iii) $\angle BCD = 90°$ (rt. $\angle$ in the semicircle)

(iv) $\angle BCA = 61°$ ($\angle$s in the alt. segment)

(v) $\angle PAD = 29°$ ($\angle$s in the alt. segment)

b) Cyclic quadrilateral.

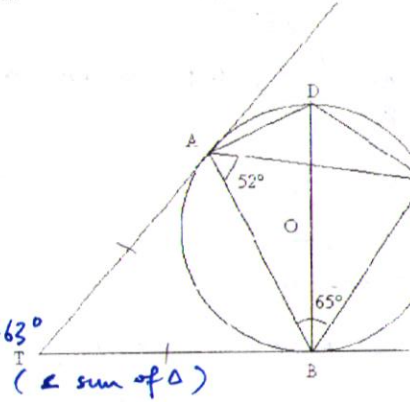
3(a) $\angle ADC = 180° - 65°$ (opp. $\angle$s in a cyclic quad.)
$= 115°$

(b) $\angle ACB = 180° - 52° - 65°$ ($\angle$ sum of Δ)
$= 63°$

$\angle ADB = 63°$ ($\angle$s in the same segment)

3. In the figure below, BD is a diameter of a circle passing through the points A, B, C and D. AT and BT are tangents to the circle at A and B respectively. $\angle BAC$ is $52°$ and $\angle ABC$ is $65°$. O is the centre of the circle.

Find
(a) $\angle ADC$
(b) $\angle ADB$
(c) $\angle DAC$
(d) $\angle DBA$
(e) $\angle ATB$
(f) the radius of the circle, given that $BT = 7$ cm



c) $\angle DAC = 90° - 52°$
$= 38°$

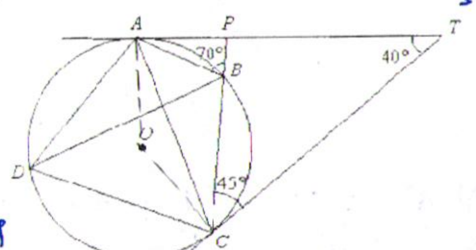
d) $\angle DBA = 180° - 90° - 63°$ ($\angle$ sum of Δ)
$= 27°$

e) $B\hat{A}T = 63°$ ($\angle$s in alt. segment)
$A\hat{B}T = 63°$ (base $\angle$s of isos. Δ)
$A\hat{T}B = 180° - 63° - 63°$ ($\angle$ sum of Δ)
$= 54°$

f) $\tan 27° = \frac{OB}{7}$
$OB = 3.566$
$= 3.57$ cm (correct to 3 sig. fig.)

4. The points A, B, C and D lie on a circle, centre O. TA and TC are tangents to the circle. The line CB produced meets AT at P. Given that $A\hat{B}P = 70°$, $A\hat{T}C = 40°$ and $T\hat{C}P = 45°$, giving your reasons, find

(a) $P\hat{C}A$,
(b) $A\hat{D}B$,
(c) $A\hat{O}C$.



(a) $A\hat{C}T = (180° - 40°) \div 2$ (base $\angle$s of isos. Δ)
$= 70°$

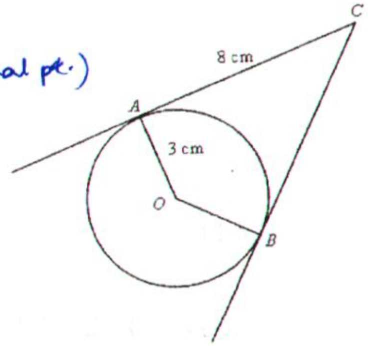
$P\hat{C}A = 70° - 45°$
$= 25°$

(b) $A\hat{D}B = P\hat{C}A$ ($\angle$s in the same segment)
$= 25°$

c) $A\hat{D}C = 70°$ (ext. $\angle$ in a cyclic quad.)

$A\hat{O}C = 70° \times 2$ ($\angle$ at centre $= 2\angle$ at circumference)
$= 140°$

5.
The diagram shows a circle with centre O and radius 3 cm. If CA and CB are tangents to the circle at A and B respectively and AC = 8 cm, find
(a) the length of BC,
(b) the length of OC,
(c) the area of the quadrilateral $AOBC$.

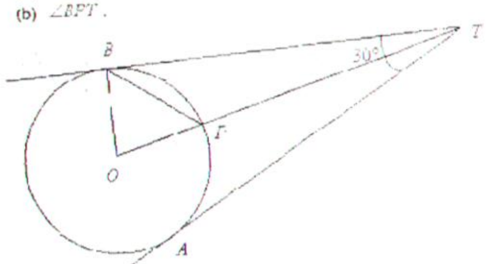


(a) $BC = AC$ (tangents from an external pt.)
$= 8 cm$

(b) $OC = \sqrt{8^2+3^2}$
$= 8.544$
$= 8.54 cm$

(c) Area of $AOBC = 2\left(\frac{1}{2} \times 8 \times 3\right)$
$= 24 cm^2$.

6. In the diagram, AT and BT are tangents to the circle, with the centre O and $\angle ATB = 30°$.
Find
(a) $\angle BOT$,
(b) $\angle BPT$.



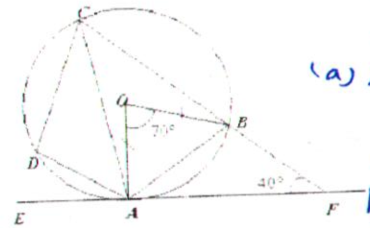
a) $B\hat{T}O = 30° \div 2$ (tangents from an external pt.)
$= 15°$

$B\hat{O}T = 180° - 90° - 15°$ ($\angle$ sum of Δ)
$= 75°$

(b) $\angle OBP = (180° - 75°) \div 2$
$= 52.5°$

$\angle BPT = 180° - 52.5°$ (adj. $\angle$s on a str. line)
$= 127.5°$

7. In the diagram, O is the center of the circle. $ABCD$ is a cyclic quadrilateral and EAF is a tangent to the circle at A. Given that $\angle AOB = 70°$, and $\angle AFB = 40°$, stating your reasons clearly, calculate
(a) $\angle BAF$
(b) $\angle ADC$.



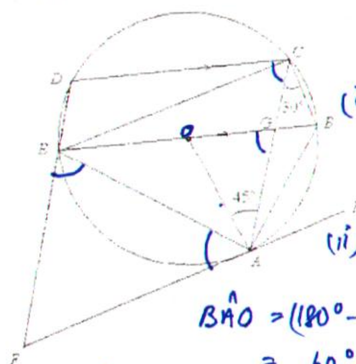
(a) $\angle ACB = 70° \div 2$ ($\angle$ at centre = 2 $\angle$ at circumference)
$= 35°$

$\angle BAF = 35°$ ($\angle$s in alt. segment)

b) $\angle CAF = 180° - 40° - 35°$ ($\angle$ sum of Δ)
$= 105°$

$\angle CAF$ ($\angle$s in alt. segment)
$= 105°$

8. In the diagram below, A, B, C, D and E are points on the circle with center O. DC is parallel to the diameter EB of the circle with center O.
(a) Given that $\angle OAC = 45°$, $\angle ACB = 30°$, DEF is a straight line and FAH is a tangent to the circle, calculate,
(i) $\angle AOB$,
(ii) $\angle BAC$,
(iii) $\angle AEF$.
(b) Prove that ΔAEF is similar to ΔCAE.



(i) $O\hat{E}A = 30°$ ($\angle$s in the same segment)
$A\hat{O}E = 180° - 30° - 30°$ (base $\angle$s of isos. Δ)
$= 120°$

(ii) $A\hat{O}B = 180° - 120°$ (adj. $\angle$s on a str. line)
$= 60°$

$B\hat{A}O = (180° - 60°) \div 2$ (base $\angle$s of isos. Δ)
$= 60°$

$B\hat{A}C = 60° - 45°$
$= 15°$

$\angle AEF = 75°$ (ext. $\angle$ of a cyclic quad.)

(iii) $O\hat{C}A = 180° - 60° - 45°$ ($\angle$ sum of Δ)
$= 75°$
$A\hat{C}D = 75°$ (corr. $\angle$s)

Answers:
1a) DODC = 49° (b) DCED = 41° (c) DAEC = 84° (d) x = 75.5°
2(a) (i) 58° (ii) 29° (iii) 90° (iv) 61° (v) 29° b) Cyclic Quadrilateral
3a) 115° 3b) 63° 3c) 38° 3d) 27° 3e) 54°
4a) 25° 4b) 25° 4c) 140° 5a) BC=8cm 5b) 8.54cm 5c) 24 cm²
6a) 75° 6b) 127.5° 7a) 35° 7b) 105° 8(a) (i) 120° (ii) 15° (iii) 75°

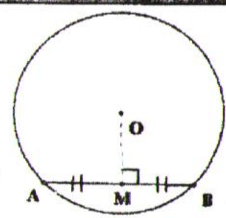
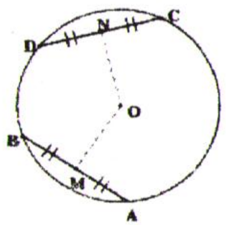
(b) $\angle ACE = 60°$ ($\angle$ at centre = 2 $\angle$ circumference)

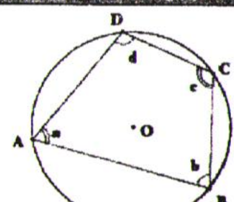
$\angle EAF = 60°$ ($\angle$s in the alt. segment)

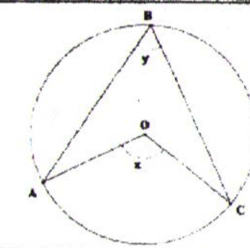
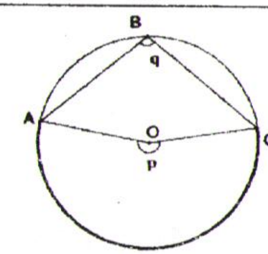
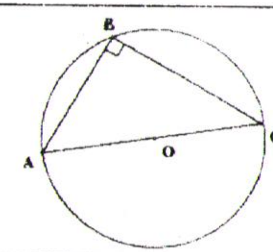
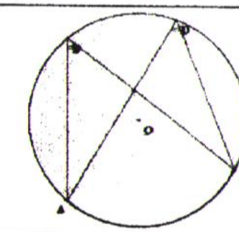
$\angle AEF = \angle CAE$
$= 75°$
$\angle EAF = \angle ACE$
$= 60°$
$\therefore$ ΔAEF is similar to ΔCAE (AAA similarity)

Properties of Circles (Angle Properties)

2 symmetrical properties of Circles:

Symmetry Property		
Perpendicular line drawn from centre of circle O bisects the chord AB (⊥ from centre bisects chord)		i. If OM ⊥ AB, then AM = BM ii. If AM = BM, then OM ⊥ AB. iii. OA = OB (radii)
Equal chords are equidistant from the centre. (equal chords, equidistant from centre)		i. If AB = CD, then OM = ON ii. If OM = ON, then AB = CD

Angle Property		
Angles in opposite segments of a circle are supplementary (add up to 180°) (opp ∠s of cyclic quad)		i. $\angle a + \angle c = 180°$ ii. $\angle b + \angle d = 180°$
Exterior angle of cyclic quad is equal to the interior opposite angle. (ext. ∠s of cyclic quad		$\angle x = \angle y$

Angle Property		
An angle at the centre of the circle is twice any angle at the circumference, subtended by the same arc. (∠ at centre = 2 ∠ at circumference)		i. $\angle x = 2\angle y$
		ii. $\angle p = 2\angle q$
An angle at the circumference subtended by the diameter of a circle is a right angle. (rt ∠ in a semi-circle)		AC = Diameter $\angle ABC = 90°$
Angles in the same segment of a circle are equal. (∠s in the same segment)		$\angle ABC = \angle ADC$