West Spring Secondary School
2011 3E EM Assignment 28: Properties of Circles

Name: _____ ( ) **Class:** _____ **Date:** _____

Recap
Previously, we learned the following:

1. Symmetrical Properties of Circles
   a. Perpendicular bisecting chord
   b. Equal chords are equidistant from centre
2. Angle Properties of Circles:
   a. ∠ at centre = twice at circumference
   b. ∠ in a semicircle
   c. ∠ in the same segment

Today, we will continue with other angle properties involving circles.

Overview
This worksheet covers the following:

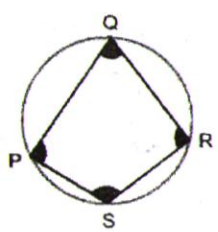
1. Angle Properties of Cyclic Quadrilaterals:
   a. Opposite ∠s of cyclic quadrilaterals
   b. Exterior ∠s of cyclic quadrilaterals

Introduction
What are cyclic quadrilaterals?

A cyclic quadrilateral is a quadrilateral (i.e. 4-sided figure) with its four vertices lying on the circumference of a circle.

PQRS is a cyclic quadrilateral.

The points P, Q, R and S are said to be concyclic.

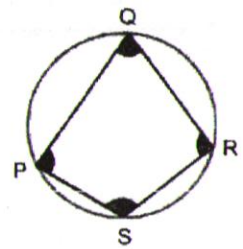


Property 1: Opposite angles of cyclic quad. are supplementary

$\hat{P}$ and $\hat{R}$ are opposite angles, $\hat{Q}$ and $\hat{S}$ are opposite angles.

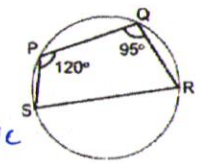
In a cyclic quadrilateral, the opposite angles are supplementary. i.e. they add up to two right angles ($180°$).

$$\hat{P}+\hat{R}=180°$$
$$\hat{Q}+\hat{S}=180°$$



Practice

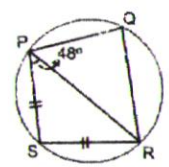
1. PQRS is a cyclic quadrilateral. Find $Q\hat{R}S$ and $P\hat{S}R$.



$\angle QRS = 180° - 120°$ (opp. ∠s of a cyclic quad.)
$= 60°$

$\angle PSR = 180° - 95°$ (opp. ∠s of a cyclic quad.)
$= 85°$

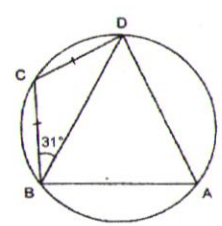
2. PQRS is a cyclic quadrilateral. Find $P\hat{Q}R$



$P\hat{S}R = 180° - 48° - 48°$ (base ∠s of isos. △)
$= 84°$

$P\hat{Q}R = 180° - 84°$ (opp. ∠s of a cyclic quad.)
$= 96°$

3. In cyclic quadrilateral ABCD, find ∠BAD



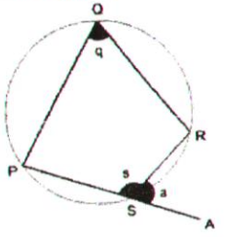
$B\hat{C}D = 180° - 31° - 31°$ (base ∠s of isos. △)
$= 118°$

$B\hat{A}D = 180° - 118°$ (opp. ∠s of a cyclic quad.)
$= 62°$

Answers:
1) 60°, 85° 2) 96° 3) 62°

Property 2: Exterior angle of cyclic quadrilateral

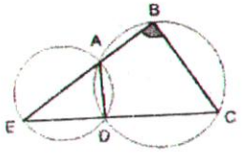
If one side of a cyclic quadrilateral is produced, the exterior angle so formed is equal to the interior opposite angle. (ext. ∠ of a cyclic quad.)



$P\hat{Q}R = R\hat{S}A$

Practice

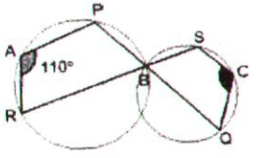
1. In the figure, EAB and EDC are straight lines. If EA is the diameter of the circle AED, find $A\hat{B}C$.



$A\hat{D}E = 90°$ (rt. ∠ in a semicircle)

$A\hat{B}C = 90°$ (ext. ∠ of cyclic quad.)

2. In the figure, PBQ and RBS are straight lines. Find $Q\hat{C}S$.

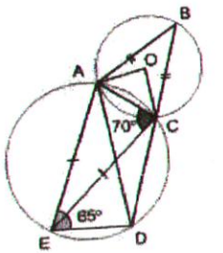


$\angle PBS = \angle PAR$ (ext. ∠ of cyclic quad.)
$= 110°$

$Q\hat{C}S = P\hat{B}S$ (ext. ∠ of cyclic quad.)
$= 110°$

Answers
1) 90° 2) 110°

3. In the figure, BCD is a straight line and the centre of the smaller circle is marked as O.
Find $A\hat{C}B$, $A\hat{B}C$, $A\hat{O}C$ and $A\hat{D}C$.



$A\hat{C}B = A\hat{E}D$ (ext. ∠ of cyclic quad.)
$= 65°$

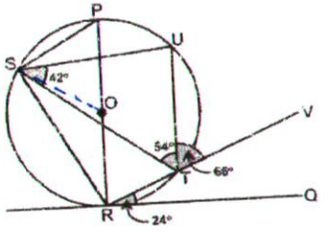
$A\hat{B}C = 180° - 65° - 65°$ (base ∠s of isos. Δ)
$= 50°$

$A\hat{O}C = 50° \times 2$ (∠ at centre = 2 ∠ at circumference)
$= 100°$

$A\hat{E}C = 180° - 70° - 70°$ (base ∠s of isos. Δ)
$= 40°$

$\angle ADC = 40°$ (∠s in the same segment)

4. PR is a diameter of the circle with centre O.
$U\hat{S}T = 42°$, $U\hat{T}S = 54°$, $U\hat{T}V = 66°$ and $T\hat{R}Q = 24°$.
Find
(i) $S\hat{R}T$
(ii) $R\hat{S}T$
(iii) $S\hat{O}P$



(i) $S\hat{U}T = 180° - 42° - 54°$ (∠ sum of Δ)
$= 84°$

$S\hat{R}T = 180° - 84°$ (opp. ∠s of a cyclic quad.)
$= 96°$

(ii) $R\hat{S}T = U\hat{R}T$ (∠s in the alt. segment)
$= 24°$

(iii) $S\hat{T}R = 180° - 54° - 66°$ (adj. ∠s on a str. line)
$= 60°$

$S\hat{P}O = S\hat{T}R$ (∠s in the same segment)
$= 60°$

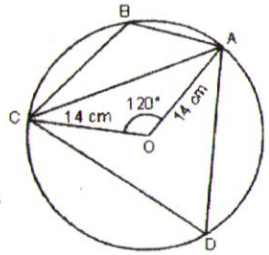
$S\hat{O}P = 180° - 60° - 60° = 60°$ (∠ sum of Δ)

Answers:
3) 65°, 50°, 100°, 40° 4i) 96° 4ii) 24° 4iii) 60°

Homework

1. A, B, C and D are four points on a circle with centre O. Given that $\angle AOC = 120°$, calculate

a) $\angle ADC$
b) $\angle ABC$
c) $\angle OAC$

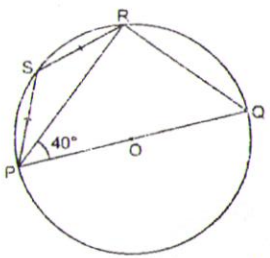


(a) $\angle ADC = 120° \div 2$ ($\angle$ at centre = 2 $\angle$ at circumference)
$= 60°$

(b) $\angle ABC = 180° - 60°$ (opp. $\angle$s of a cyclic quad.)
$= 120°$

c) $\angle OAC = (180° - 120°) \div 2$ (base $\angle$s of isos. $\triangle$)
$= 30°$

2.



In the figure, PQRS is a cyclic quadrilateral.
PS = SR and PQ is a diameter.
Find $\angle PRS$

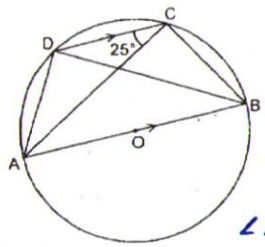
$P\hat{R}Q = 90°$ (rt. $\angle$ in a semicircle)

$P\hat{Q}R = 180° - 90° - 40°$ ($\angle$ sum of $\triangle$)
$= 50°$

$P\hat{S}R = 180° - 50°$ (opp. $\angle$s of a cyclic quad.)
$= 130°$

$P\hat{R}S = (180° - 130°) \div 2$ (base $\angle$s of isos. $\triangle$)
$= 25°$

3.



AB is a diameter of the circle.
Given DC || AB and $\angle DCA = 25°$.
find $\angle ADC$

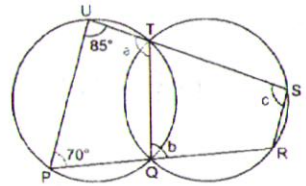
$\angle ADB = 90°$ (rt. $\angle$ in a semicircle)

$\angle BAC = \angle ACD$ (alt. $\angle$s)
$= 25°$

$\angle BDC = \angle BAC$ ($\angle$s in the same segment)
$= 25°$

$\angle ADC = 90° + 25°$
$= 115°$

4. In the figure below, UTS and PQR are line segments. $\angle PUT = 85°$ and $\angle UPQ = 70°$
Find a, b and c.

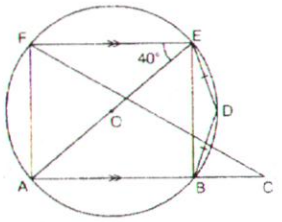


$\angle a = 180° - 70°$ (opp. $\angle$s in a cyclic quad.)
$= 110°$

$\angle b = 85°$ (ext. $\angle$ of a cyclic quad.)

$\angle c = 180° - 85°$ (opp. $\angle$s of a cyclic quad.)
$= 95°$

5.



In the figure, ABC is a straight line and EF || AB

Find ∠EDB and ∠DBC

$\hat{DBE} = (180° - 140°) \div 2$ (base ∠s of isos. Δ)

$= 20°$

$\hat{ABE} = 90°$ (rt. ∠ in a semicircle)

$\hat{DBC} = 180° - 90° - 20°$ (adj. ∠s on a str. line)

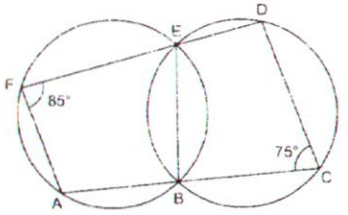
$= 70°$

$\hat{BHE} = 40°$ (alt. ∠s)

$\hat{EDB} = 180° - 40°$ (opp. ∠s of a cyclic quad.)

$= 140°$

6.



In the figure, FED and ABC are line segments

∠AFE = 85° and ∠BCD = 75°

Find ∠BAF and ∠EDC

$\hat{CBE} = 85°$ (ext. ∠ of a cyclic quad.)

$\hat{BEF} = 75°$ (ext. ∠ of a cyclic quad.)

$\hat{BHF} = 180° - 75°$ (opp. ∠s of a cyclic quad.)

$= 105°$

$\hat{EDC} = 180° - 85°$ (opp. ∠s of a cyclic quad.)

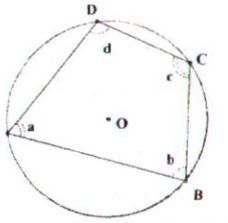
$= 95°$

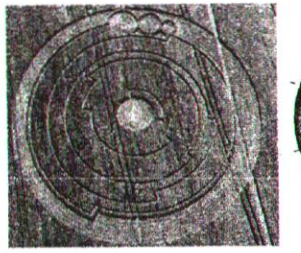
Answers:

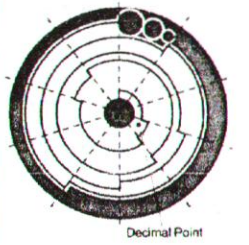
1a) 60° 1b) 120° 1c) 30° 2) 25° 3) 115° 4) a=110°, b=85°, c=95°
5) 140°, 70° 6) 105°, 95°

Summary

Complete the table below.

Angle Property		
Angles in opposite segments of a circle are supplementary (add up to 180°) (∠s in opp segment) (opp ∠s in cyclic quad)		i. $\angle a + \angle c =$ 180° ii. $\angle b + \angle d =$ 180°





Decimal Point

Math Puzzle:

Crop circles are mysterious patterns found on farmlands and fields in England and North America. The above crop circle appeared in the English countryside. It's secret has recently been deciphered. What do you think it represents? (Hint: think circles)