West Spring Secondary School
2011 3E EM Assignment 1: Properties of Circles

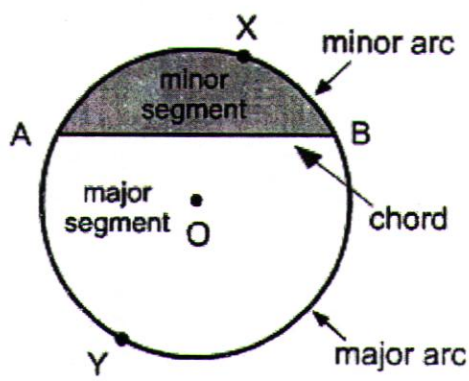
Name: _____ ( ) Class: _____ Date: _____

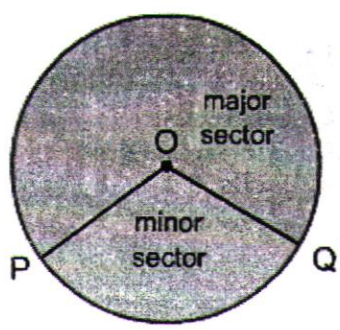
Overview

This worksheet covers the following:

1. **Introduction to parts of a circle**
2. **Symmetrical Properties of Circles**
   a. **Perpendicular bisecting chord**
   b. **Equal chords are equidistant from centre**
3. **Angle Properties of Circles:**
   a. **∠ at centre = twice at circumference**
   b. **∠ in a semicircle**
   c. **∠ in the same segment**

Introduction to Parts of a Circle





Arc: AX, XB, AY, YB

Minor Arc: AXB

Major Arc: AYB

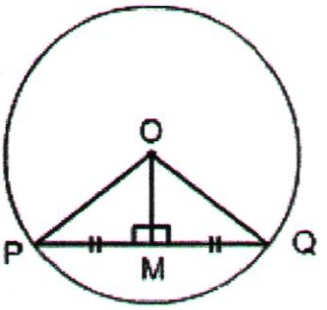
Segment: Minor segment AXB, major segment AYB

Sector: Major sector POQ, Minor sector PQO

Property 1: Perpendicular Bisecting Chord

The perpendicular bisector of a chord passes through the centre.

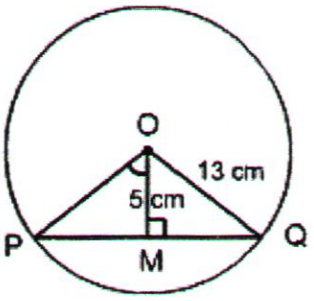
i.e. a straight line drawn from the centre of a circle to bisect a chord is perpendicular to the chord.



$PQ \perp OM$

Example 1:

Find the values of PQ and $P\hat{O}M$ in the circle.



$MQ = \sqrt{13^2 - 5^2}$

$= 12$

$PQ = 12 \times 2$

$= 24$ cm "

$\tan P\hat{O}M = \frac{12}{5}$

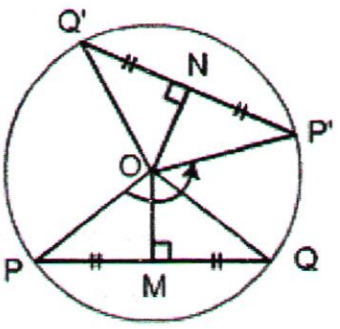
$P\hat{O}M = 67.38$

$= 67.4°$ (correct to 1 dec. pl.) "

Property 2: Equal chords are equidistant from centre of circle

In a circle, equal chords are equidistant from the centre.

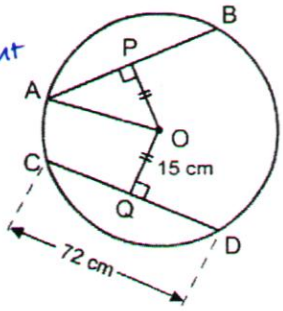
i.e. chords which are equidistant from the centre are equal



$PQ = P'Q'$ and $OM = ON$

Example:

Find the values of AP and OC in the circle.



$AP = 72 \div 2$ (equal chords are equidistant from centre of circle)

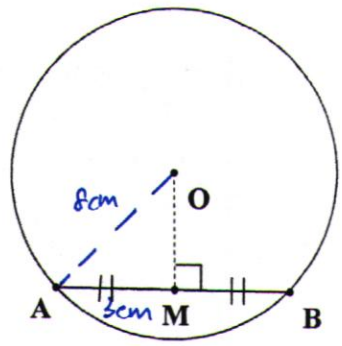
$= 36$ cm

$OC = \sqrt{36^2 + 15^2}$

$= 39$ cm "

Practice

1. In the figure, AB = 6 cm and the radius of the circle is 8 cm.



a) Find length of OM.
b) Find $\angle OAB$ if $\angle AOB = 80°$

(a) AM = 3 cm

$OM = \sqrt{8^2 - 3^2}$

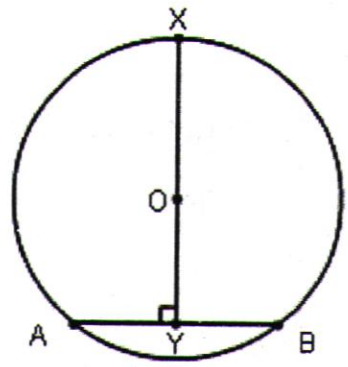
≈ 7.416

$= 7.42$ cm (correct to 3 sig. fig.)

(b) $\angle OAB = (180° - 80°) \div 2$ (base $\angle$s of isos. $\triangle$)

$= 50°$

2. In the figure, O is the centre of the circle and XY is perpendicular to the chord AB. Given that AB = 3 cm and OY= 5 cm, calculate the radius of the circle.



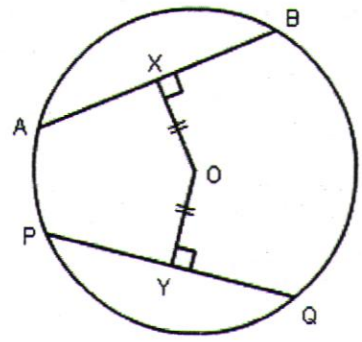
Radius $= \sqrt{5^2 + 1.5^2}$

≈ 5.220

$= 5.22$ cm (correct to 3 sig. fig.)

3. O is the centre of the circle. Given that AB = 8 cm and OX = OY = 3 cm,

a) Find the length of YQ
b) Find the radius of the circle



(a) YQ = 8 ÷ 2

= 4 cm

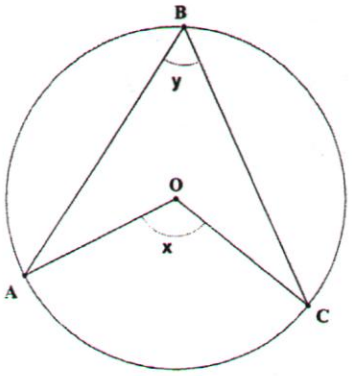
(b) Radius $= \sqrt{4^2 + 3^2}$

$= 5$ cm

Answers:
1a) 7.42 1b) 50° 2) 5.22 cm 3a) 4 cm 3b) 5 cm

Angle Property 1: ∠ at centre = twice at circumference

An angle at the centre of a circle is twice any angle at the circumference subtended by the same arc.
(∠ at centre = 2 ∠ at $\odot^{ce}$)

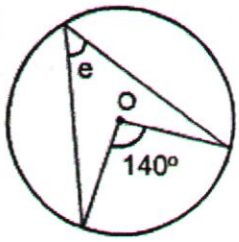


$\angle x =$ 2 ∠y

Practice:

1. Find the unknown angle marked in the circle with centre O.

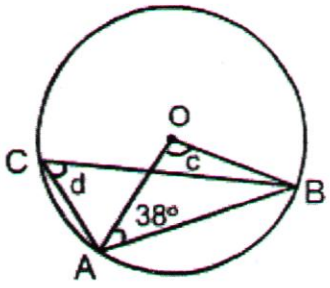
∠e = 140° ÷ 2 (∠ at centre = 2 ∠ at circumference)
= 70°



2. Find the unknown angles marked in the circle with centre O.

∠c = 180° - 38° - 38° (base ∠s of isos. Δ)
= 104°

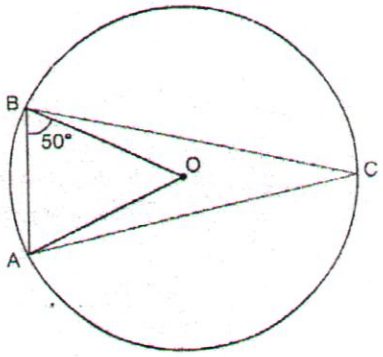
∠d = 104° ÷ 2 (∠ at centre = 2 ∠ at circumference)
= 52°



3. In the diagram below find ∠ACB.

∠ AOB = 180° - 50° - 50° (base ∠s of isos. Δ)
= 80°

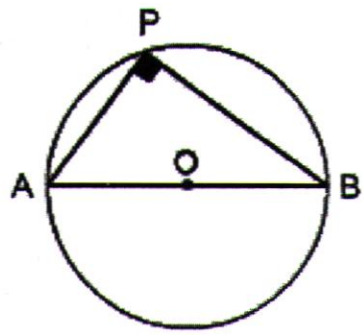
∠ACB = 80° ÷ 2 (∠ at centre = 2 ∠ at circumference)
= 40°



Answers:
1) 70° 2) 104°, 52° 3) 40°

Angle Property 2: (Angle in a semicircle)

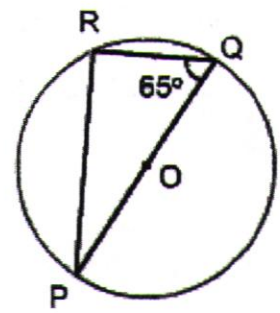
Every angle at the circumference subtended by the diameter of a circle is a right angle. **(rt. ∠ in a semicircle)**



$A\hat{P}B = 90°$

Practice

1. In the diagram, PQ is a diameter of the circle.
If $P\hat{Q}R = 65°$, calculate $R\hat{P}Q$.

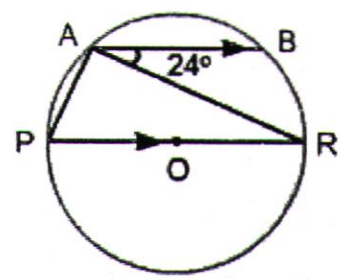


$P\hat{R}Q = 90°$ (rt. ∠ in a semicircle)

$R\hat{P}Q = 180° - 90° - 65°$ (∠ sum of △)

$= 25°$

2. In the diagram, POR is parallel to AB and $B\hat{A}R = 24°$.
Find $A\hat{P}R$.



$P\hat{R}A = B\hat{A}R$ (alt. ∠s, AB // PR)

$= 24°$

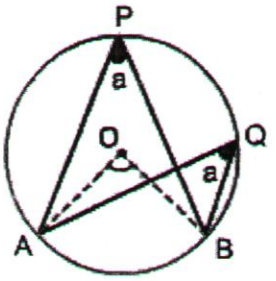
$P\hat{A}R = 90°$ (rt. ∠ in a semicircle)

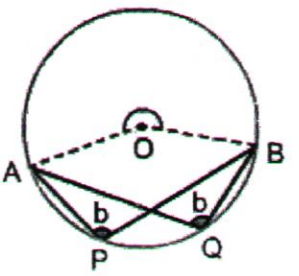
$A\hat{P}R = 180° - 90° - 24°$ (∠ sum of △)

$= 66°$

Angle Property 3: (Angles in the same segment)

Angles in the same segment of a circle are equal. (∠s in the same segment)

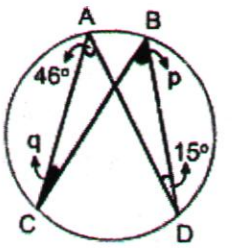




$A\hat{P}B = A\hat{Q}B$

Practice

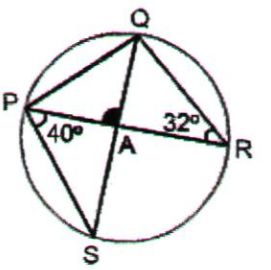
1. Find the unknown angles marked in the diagram.



$\angle p = 46°$, (∠s in the same segment)

$\angle q = 15°$ (∠s in the same segment)

2. In the diagram, PR intersects QS at A. Find $P\hat{A}Q$.

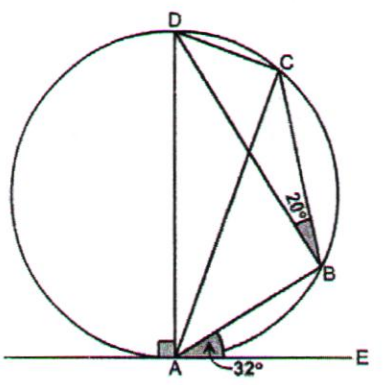


$AQR = 40°$ (∠s in the same segment)

$P\hat{A}Q = 32° + 40°$ (ext. ∠ = opp. interior ∠s)

$= 72°$ //

4. AD is a diameter of the circle and perpendicular to AE. Given that $B\hat{A}E = 32°$ and $C\hat{B}D = 20°$, calculate $A\hat{D}C$.



$C\hat{A}D = 20°$ (∠s in the same segment)

$A\hat{C}D = 90°$ (∠ in a semicircle)

$A\hat{D}C = 180° - 90° - 20°$ (∠ sum of △)

$= 70°$ //

Answers:
1) 46°, 15° 2) 72° 3) 70°

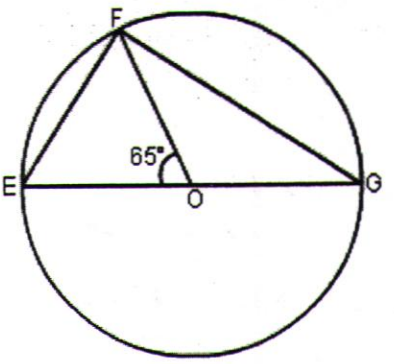
Homework

(State all reasons clearly)

1. In the diagram, O is the centre of the circle. Given that $\angle EOF = 65°$, find

   a) $\angle OEF$

   b) $\angle OFG$



(a) $\angle OEF = (180° - 65°) \div 2$ (base $\angle$s of isos. $\triangle$)

$= 57.5°$ //

(b) $E\hat{F}G = 90°$ (rt. $\angle$ in a semicircle)

$OFG = 90° - 57.5°$

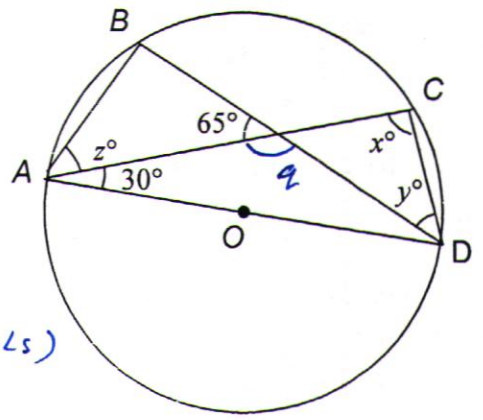
$= 32.5°$ //

2. In the figure below, points *A*, *B*, *C* and *D* all lie on the circumference of the circle with center *O*. *AD* is the diameter of the circle. Find

   (a) $x°$

   (b) $y°$

   (c) $z°$



(a) $\angle x = 90°$ (rt. $\angle$ in a semicircle) //

(b) $\angle q = 180° - 65°$ (adj. $\angle$s on a str. line)

$= 115°$

$\angle y = 115° - 90°$ (ext. $\angle$ = opp. interior $\angle$s)

$= 25°$ //

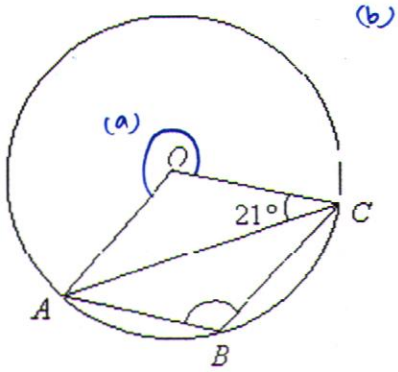
(c) $\angle z = \angle y$ ($\angle$s in the same segment)

$= 25°$ //

3. A circle with centre *O* passes through *A*, *B* and *C* and $\angle OCA = 21°$.

   (a) Mark with an arc the reflex angle *AOC*.

   (b) Find $\angle ABC$.



(b) $A\hat{O}C = 180° - 21° - 21°$ (base $\angle$s of isos. $\triangle$)

$= 138°$

reflex $A\hat{O}C = 360° - 138°$ ($\angle$s at a point)

$= 222°$

$A\hat{B}C = 222° \div 2$ ($\angle$ at centre = 2 $\angle$ at circumference)

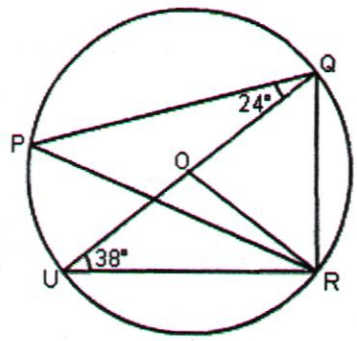
$= 111°$ //

4. In the diagram, O is the centre of the circle. ∠OQP = 24° and ∠OUR = 38°. Find

a) $\angle QRU$

b) $\angle RPQ$

c) $\angle ROQ$

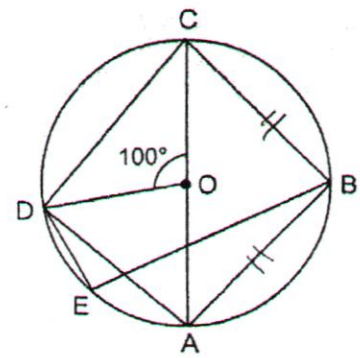


(a) ∠QRU = 90° (rt. ∠ in a semicircle)

(b) ∠RPQ = 38° (∠s in the same segment)

(c) ∠ROQ = 38° × 2 (∠ at centre = 2 ∠ at circumference)
= 76°

5. O is the centre of the circle. AC is the diameter.
Given that BC = AB, and ∠COD = 100°, find ∠BED.



$O\hat{C}D$ = (180° − 100°) ÷ 2 (base ∠s of isos. Δ)
= 40°

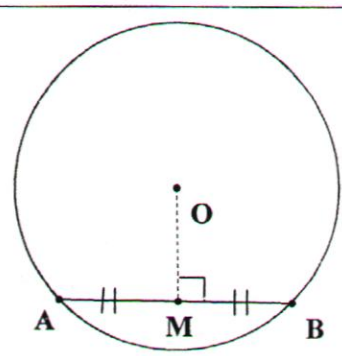
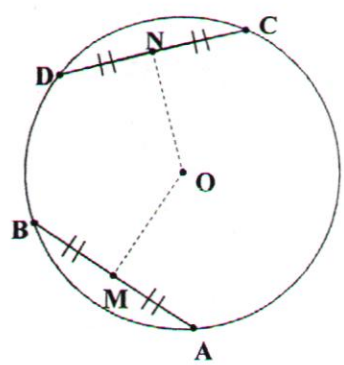
$A\hat{C}B$ = (180° − 90°) ÷ 2 (base ∠s of isos. Δ)
= 45°

$B\hat{E}D$ = 180° − 40° − 45° (opp. ∠s of a cyclic quad.)
= 95°

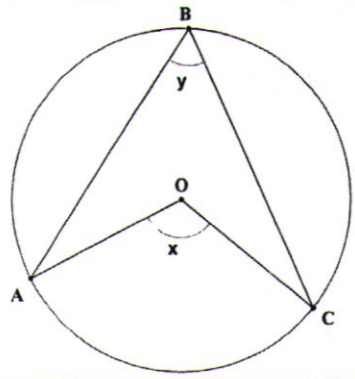
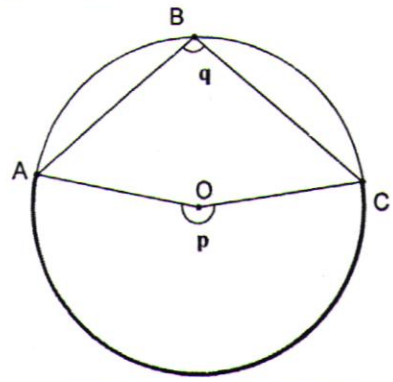
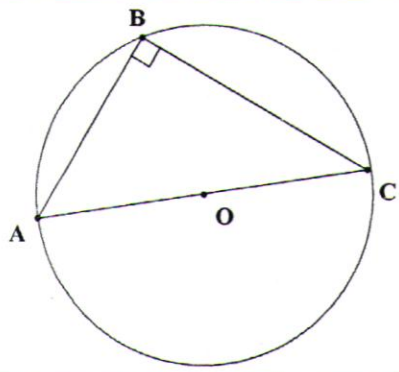
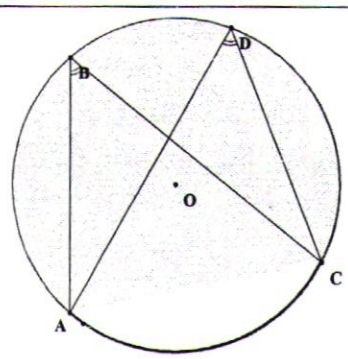
Summary

Complete the table:

Symmetry Properties of a Circle

Symmetry Property		
Perpendicular line drawn from centre of circle O bisects the chord AB **(⊥ from centre bisects chord)**		i. If OM ⊥ AB, then AM = BM ii. If AM = BM, then OM ⊥ AB. iii. OA = OB
Equal chords are equidistant from the centre. **(equal chords, equidistant from centre)**		i. If AB = CD, then OM = ON ii. If OM = ON, then AB = CD

Angle Properties of a Circle

Angle Property		
An angle at the centre of the circle is twice any angle at the circumference, subtended by the same arc. **(∠ at centre = 2 ∠ at circumference)**		i. $\angle x =$ 2∠y
		ii. $\angle p =$ 2∠q
An angle at the circumference subtended by the diameter of a circle is a right angle. **(rt ∠ in a semi-circle)**		AC = Diameter $\angle ABC =$ 90°
Angles in the same segment of a circle are equal. **(∠s in the same segment)**		$\angle ABC =$ ∠ADC

Answers:

1a) 57.5° 1b) 32.5° 2a) x = 90° 2b) y = 25° 2c) z = 25° 3a) reflex ∠AOC=222°
3b) 111° 4a) 90° 4b) 38° 4c) 76° 5) 95°