



Name: _____ () Class: _____ Date: _____

Overview

This worksheet covers the following:

- 1. Graphs of Exponential functions

Recap

Previously, we learned about graphs of quadratic and cubic functions.

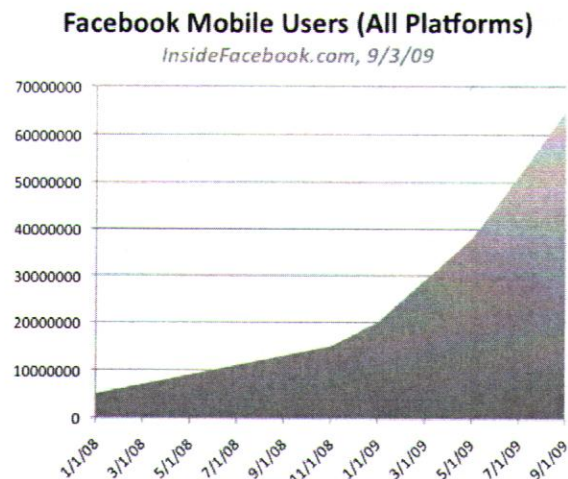
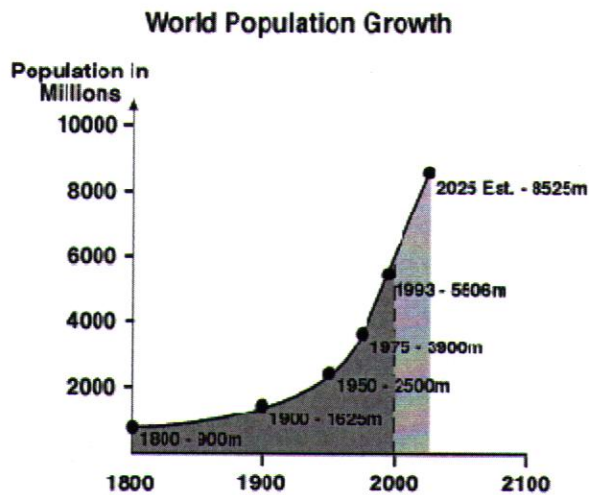
- Linear graphs ✓
- Quadratic graphs ✓
- Cubic graphs ✓
- Reciprocal graphs ✓
- Exponential graphs

Exponential Graphs

Often we read or hear in the news, “the number of *something* is growing exponentially” or “the growth of *something* is exponential”. What do they mean?

When something is increasing (growing) at an exponential rate, it means that when plotted on an x-y axis, when its x value increases a little, its y values increases a lot.

Many real world phenomena can be represented by exponential graph, e.g. world population growth, compound interest, radioactive decay.



Exponential graphs are graphs of functions which are exponential.

Let's look at what a general exponential graph looks like.

$y = a^x$ ($a > 1$)	$y = a^x$ ($0 < a < 1$)
<p>Important points to note:</p> <ul style="list-style-type: none"> As x increases, y increases exponentially 	<p>Important points to note:</p> <ul style="list-style-type: none"> As x decreases beyond $(0, 1)$ y increases exponentially

Question: Regardless of a (so long it's +ve), the graph will always pass through $(0, 1)$. True or false? Why?

Yes.

Example:

Exponential Graph: Binary Fission (Cell Division)

A bacterial cell is undergoing binary fission!

Assuming that binary fission occurs every 20 minutes, how many bacterium cells will there be at the end of the 30th 20-min block?

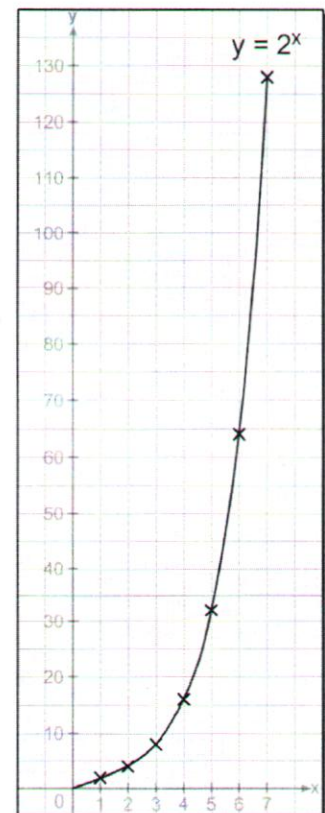
No. of 20 min block (x)	1	2	3	4	5	6	7	30
No. of bacterium cells	2	4	8	16	32	64	128	1073741824
Pattern: 2^x	2^1	2^2	2^3	2^4	2^5	2^6	2^7	2^{30}

There will be more than one billion bacterium cells!

Notice that at the end of every 20-min block, the number of cells is multiplied by 2.

Hence, the number of cells at the end of x^{th} 20-min block is 2^x .

The above relationship can be represented by the equation, $y = 2^x$.



Exponential Function, $y = ka^x$

In the scope of this lesson, the exponential function is defined as:

$$y = k a^x$$

exponent/index

base $a > 0, a \neq 1$

where k is a coefficient,

base a is a positive integer and $a \neq 1$ and

the variable x is the exponent/index.

Why can't base a be equal to 1?

Let's substitute $a = 1$ into the function: $y = k(1)^x$

$$\therefore y = k \quad \text{a constant function}$$

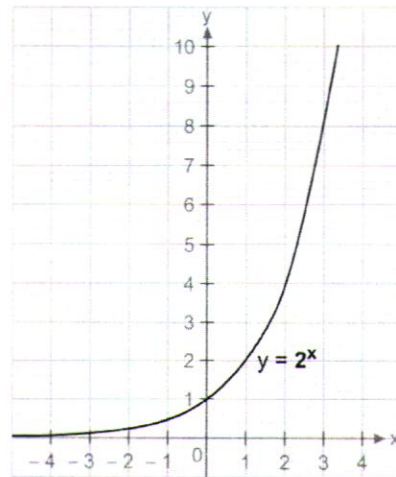
Hence, if $a = 1$, it would **disqualify** the function as an exponential function.

When $k = 1$, $y = a^x$

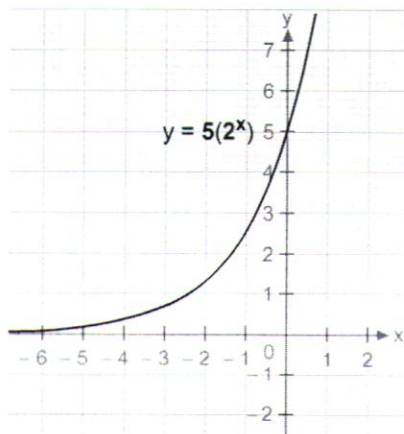
When x increases, y tends to infinity very quickly.

When x decreases, y tends to zero very quickly.

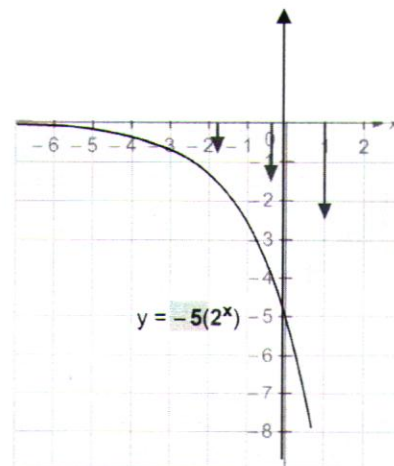
When k is positive, the graph lies above the x -axis.



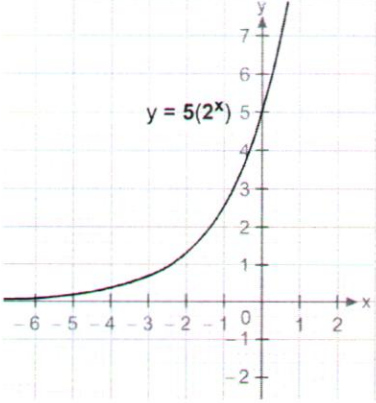
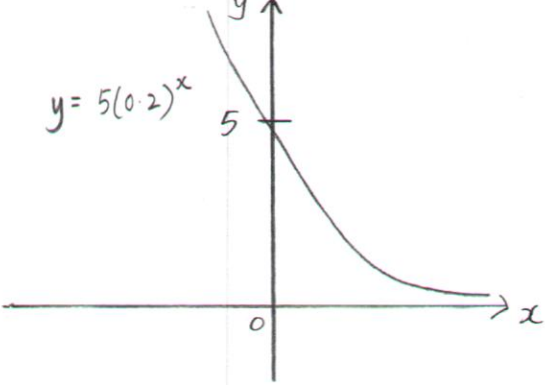
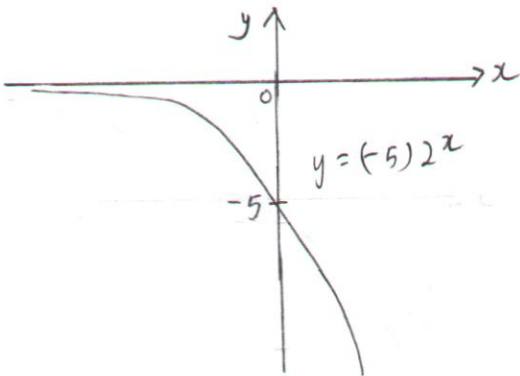
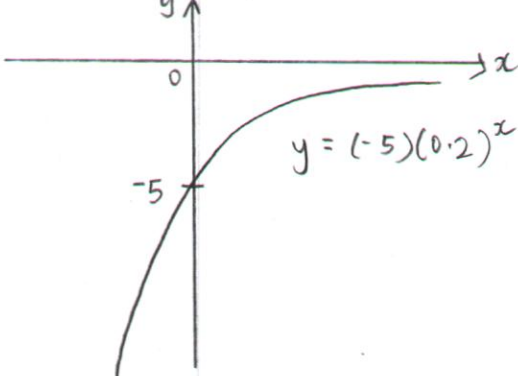
What happens when $k > 0$?
What is the y -intercept?



What happens when $k < 0$?
What is the y -intercept?



Summary

$y = ka^x$ <p>(k is positive, $a > 1$)</p>	$y = ka^x$ <p>(k is positive, $0 < a < 1$)</p>
	
$y = ka^x$ <p>(k is negative, $a > 1$)</p>	$y = ka^x$ <p>(k is negative, $0 < a < 1$)</p>
	

Practice

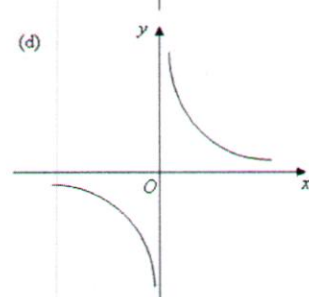
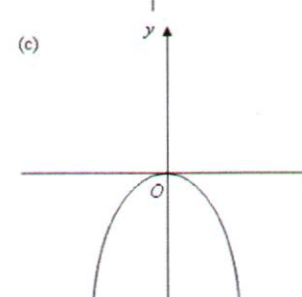
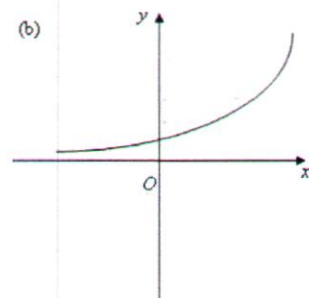
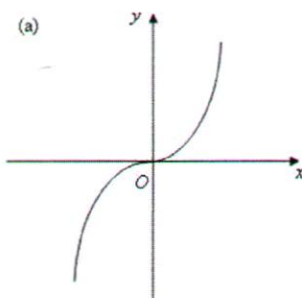
1. Match the following equations to the graphs.

(i) $y = -x^2$ (c)

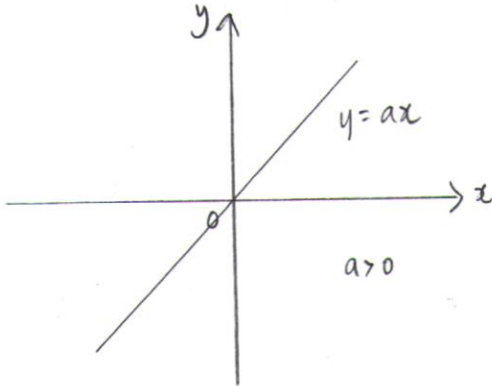
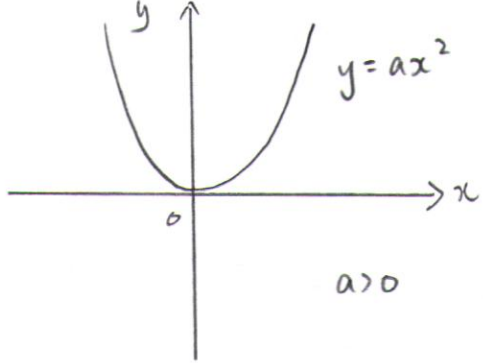
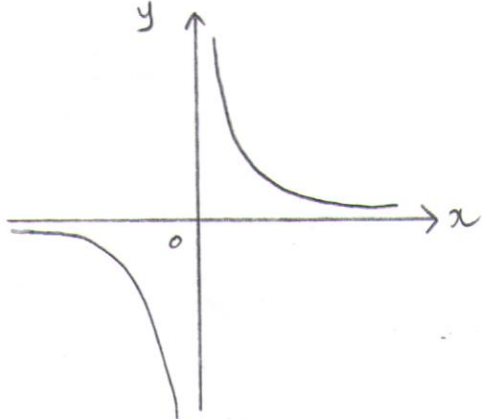
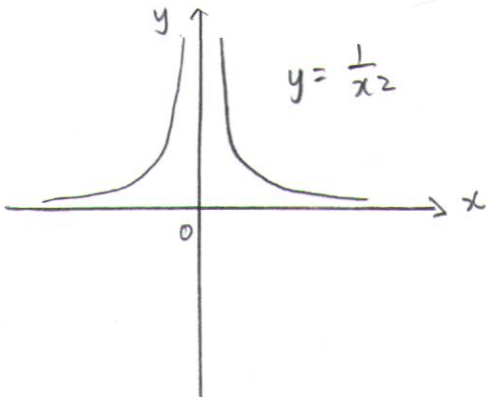
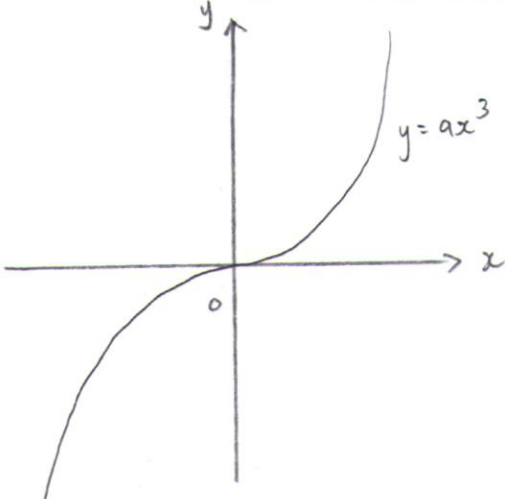
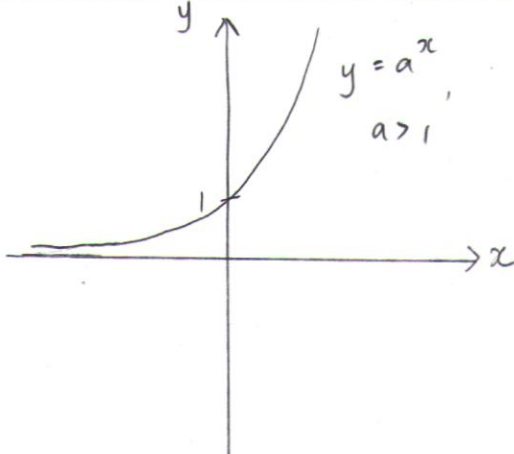
(ii) $y = x^3$ (a)

(iii) $y = \frac{1}{x}$ (d)

(iv) $y = 2^x$ (b)



2. Sketch the following functions without referring to your textbook or notes.

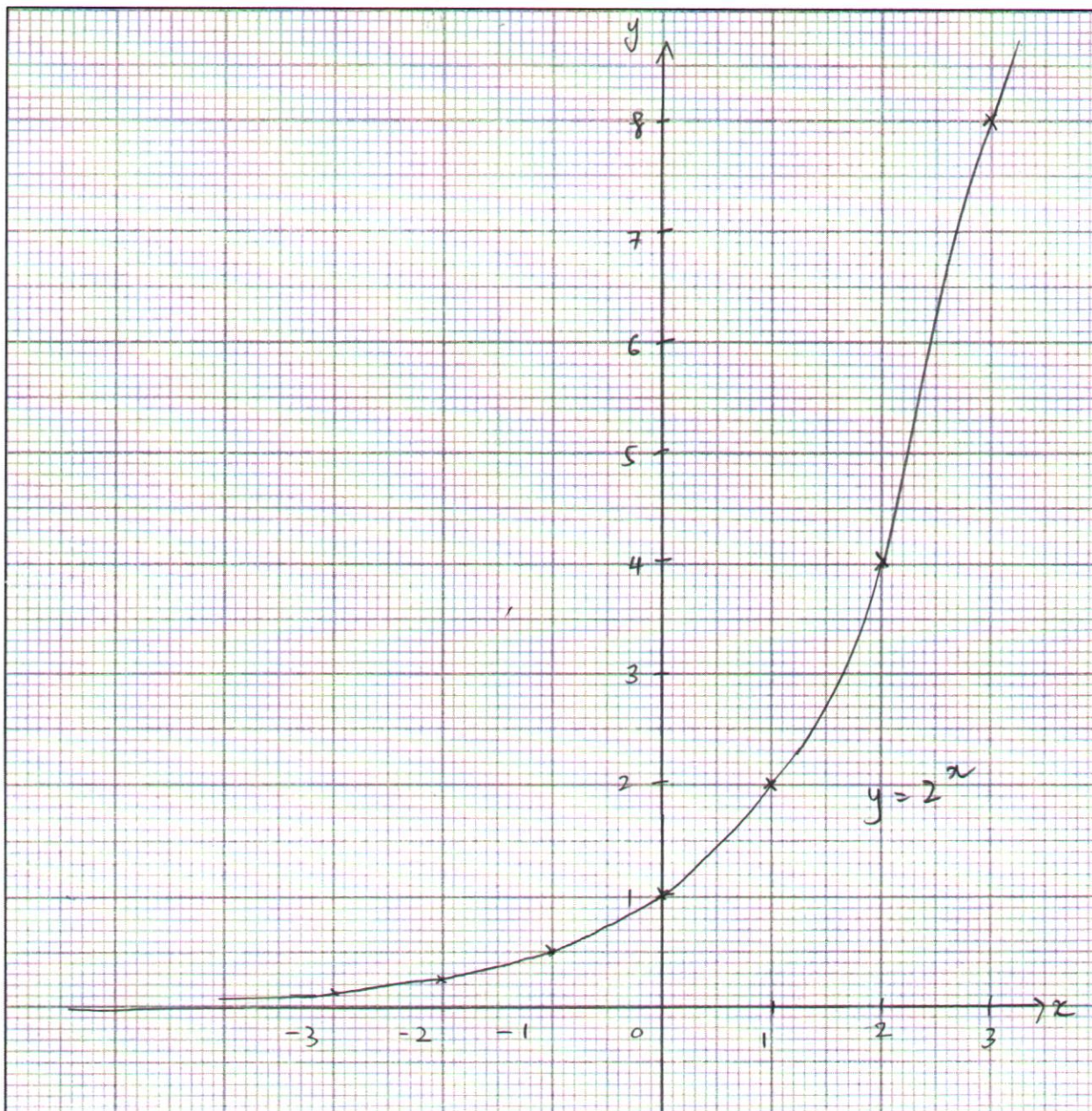
<p style="text-align: center;">$y = ax$</p> 	<p style="text-align: center;">$y = ax^2$</p> 
<p style="text-align: center;">$y = \frac{1}{x}$</p> 	<p style="text-align: center;">$y = \frac{1}{x^2}$</p> 
<p style="text-align: center;">$y = ax^3$</p> 	<p style="text-align: center;">$y = a^x \ (a > 1)$</p> 

Homework

1. (Answer using the graph provided below)

Draw the graph of $y = 2^x$ for the range $-3 \leq x \leq 3$

x	-3	-2	-1	0	1	2	3
y	0.125	0.25	0.5	1	2	4	8



2. The number of yeast cells in a cup of water at time t hours after the first observation is given by $y = 60(2^t)$.

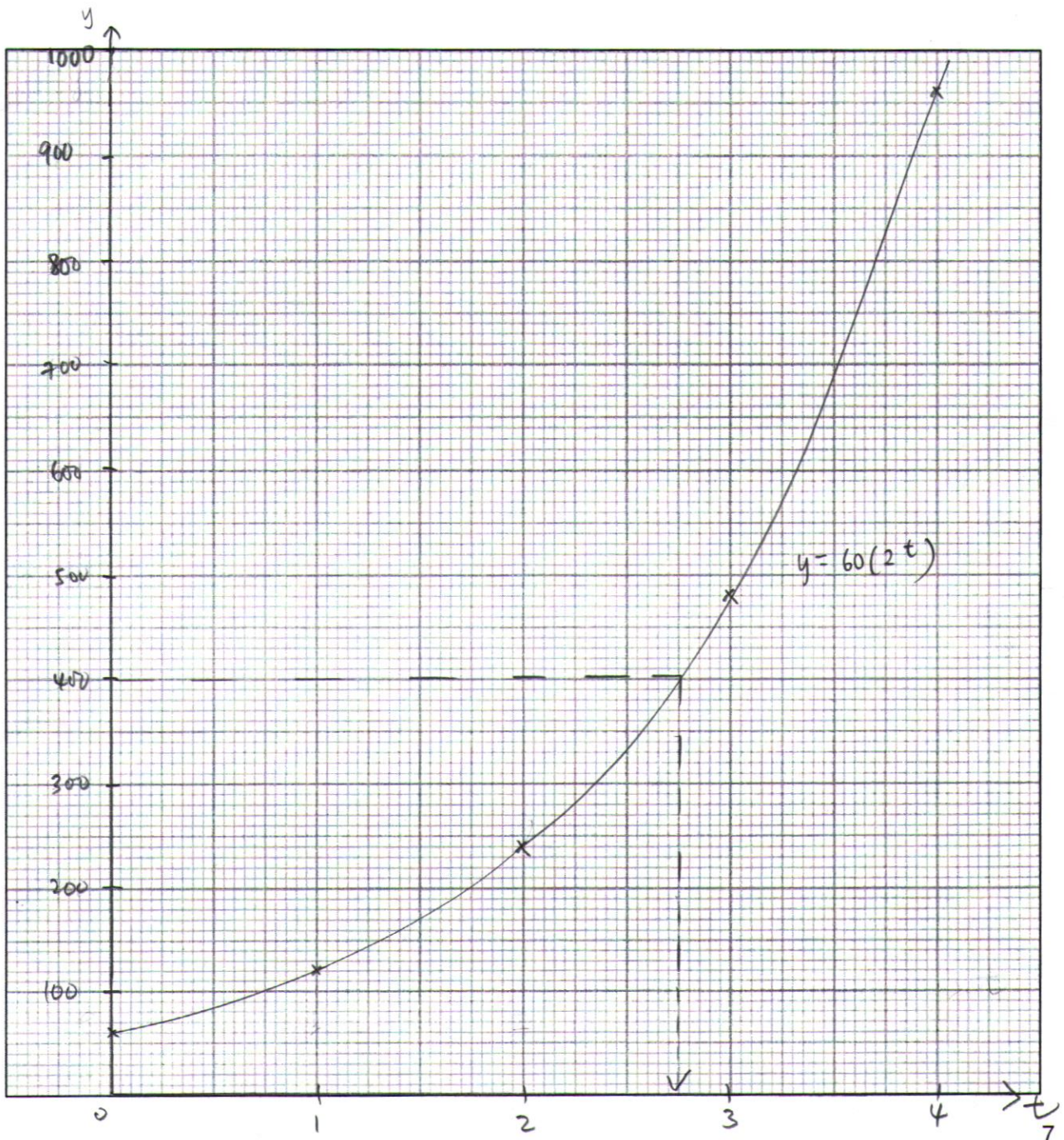
- a) Find the number of yeast cells in the cup
 i) when first observed,
 ii) 2 hours later.
- b) Draw the graph of $y = 60(2^t)$ for $0 \leq t \leq 4$
- c) Estimate the time at which the number of yeast cells is 400.

2a) i) No. of yeast = 60

ii) No. of yeast 2 hrs later = 240

c) Time = 2.75 h

t	0	1	2	3	4
y	60	120	240	480	960



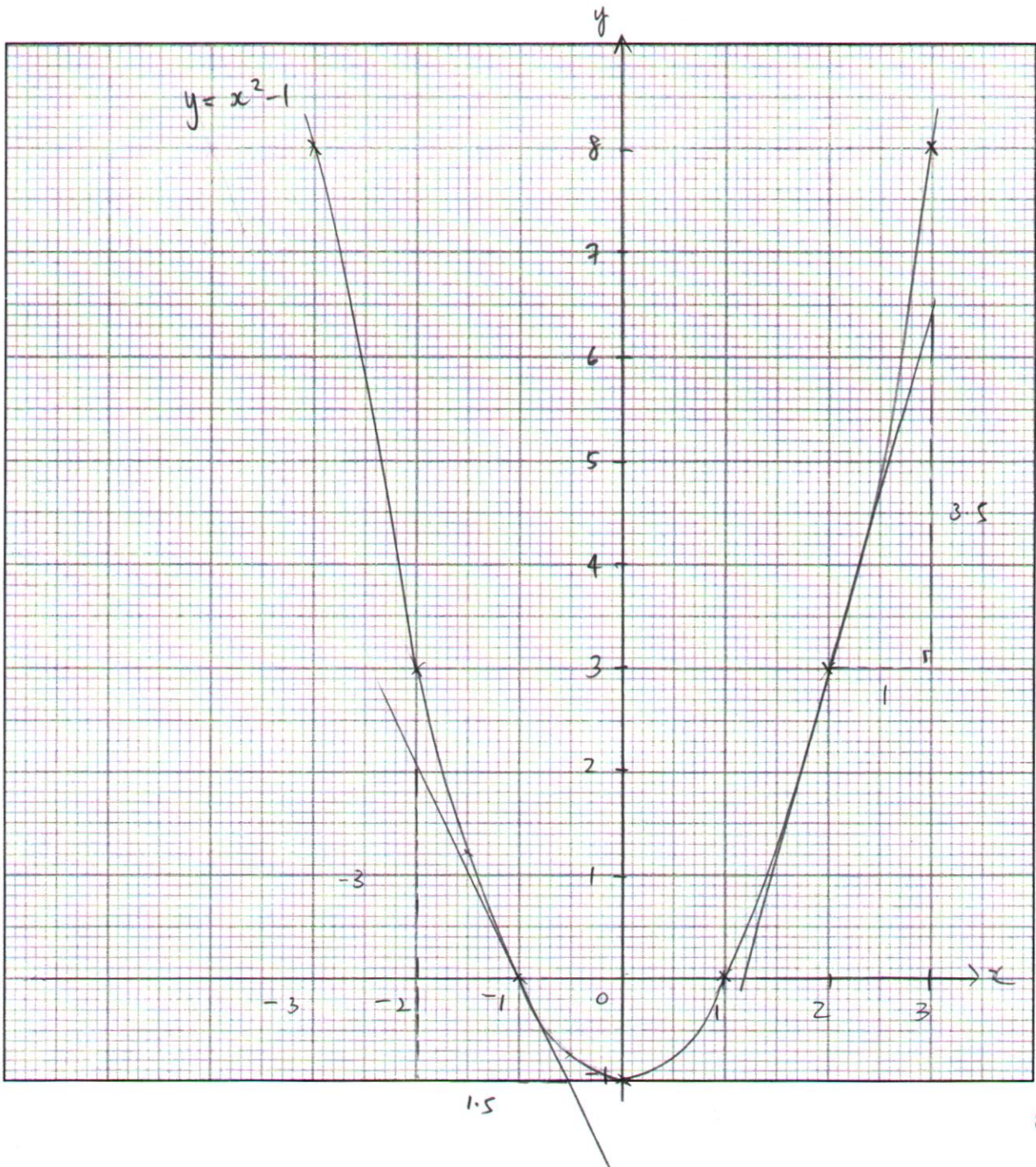
3. a) Draw the graph of $y = x^2 - 1$ for $-3 \leq x \leq 3$
 b) Find the gradient of the graph at the point where
 i) $x = -1$
 ii) $x = 0$
 iii) $x = 2$

x	-3	-2	-1	0	1	2	3
y	8	3	0	-1	0	3	8

i) Gradient at $x = -1 = \frac{-3}{1.5} = -2$ "

ii) Gradient $x = 0 = 0$ "

iii) Gradient $x = 2 = \frac{3.5}{1} = 3.5$ "



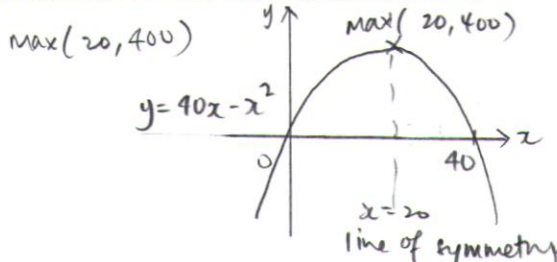
4. The revenue y (in \$) for selling x items is given by $y = 40x - x^2$

- a) Sketch the graph of $y = 40x - x^2$ for $x \geq 0$
- b) What is the maximum revenue?
- c) How many items should be sold in order to have the maximum revenue?

$$y = 40x - x^2$$

$$= x(40 - x)$$

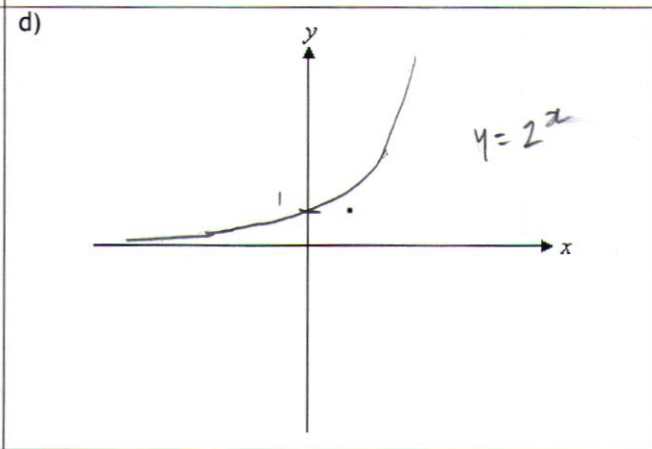
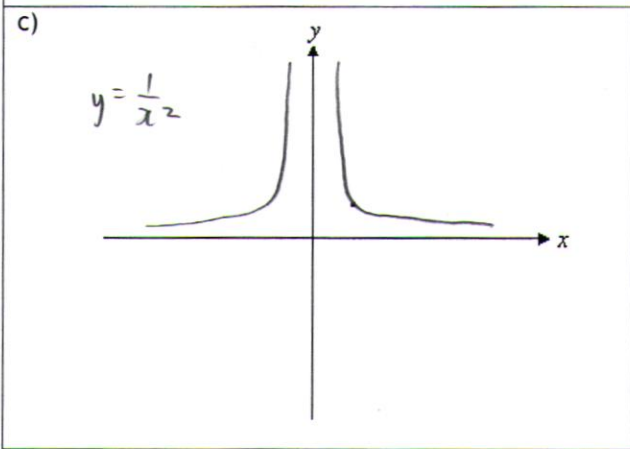
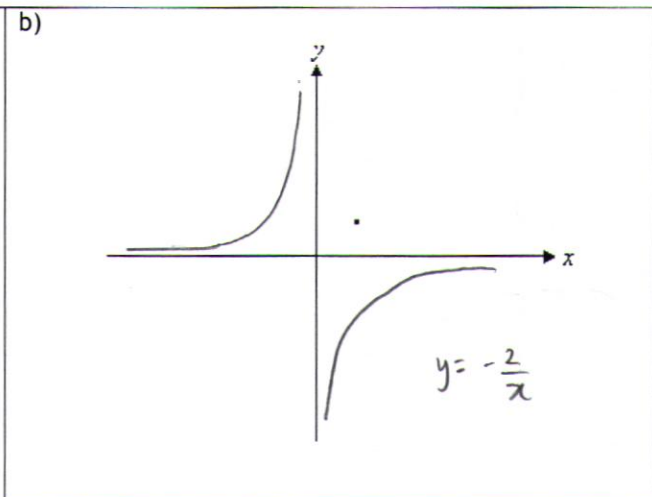
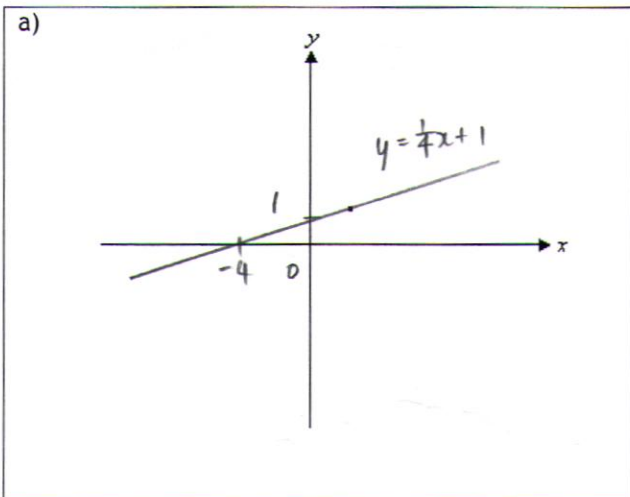
x -intercepts: $x = 0$ or $x = 40$
 line of symmetry: $x = 20$
 y -intercept: $y = 0$



b) Max. revenue
 = \$400 ,,
 c) No. of items
 = 20 ,,

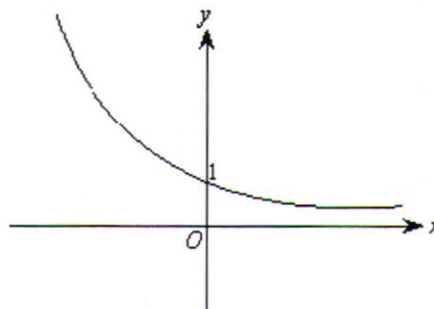
5. The point $(1, 1)$ is marked on each diagram in the answer space. On each diagram, sketch the graphs of

- (a) $y - 1 = \frac{1}{4}x$, $y = \frac{1}{4}x + 1$
- (b) $xy = -2$, $y = -\frac{2}{x}$
- (c) $y = \frac{1}{x^2}$,
- (d) $y = 2^x$.

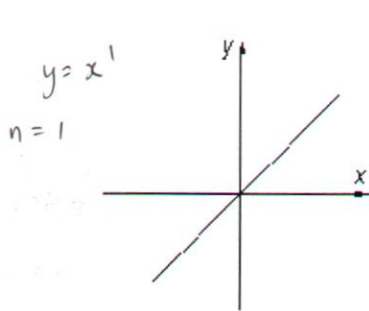


6. Write a possible equation for this graph.

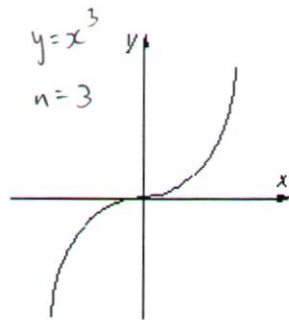
$$y = 0.5^x$$



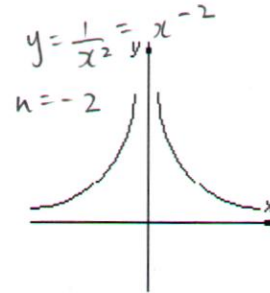
7. The equations of the three graphs are in the form of $y = x^n$. State the value or a possible value of n in each of the following graphs.



Graph (a)

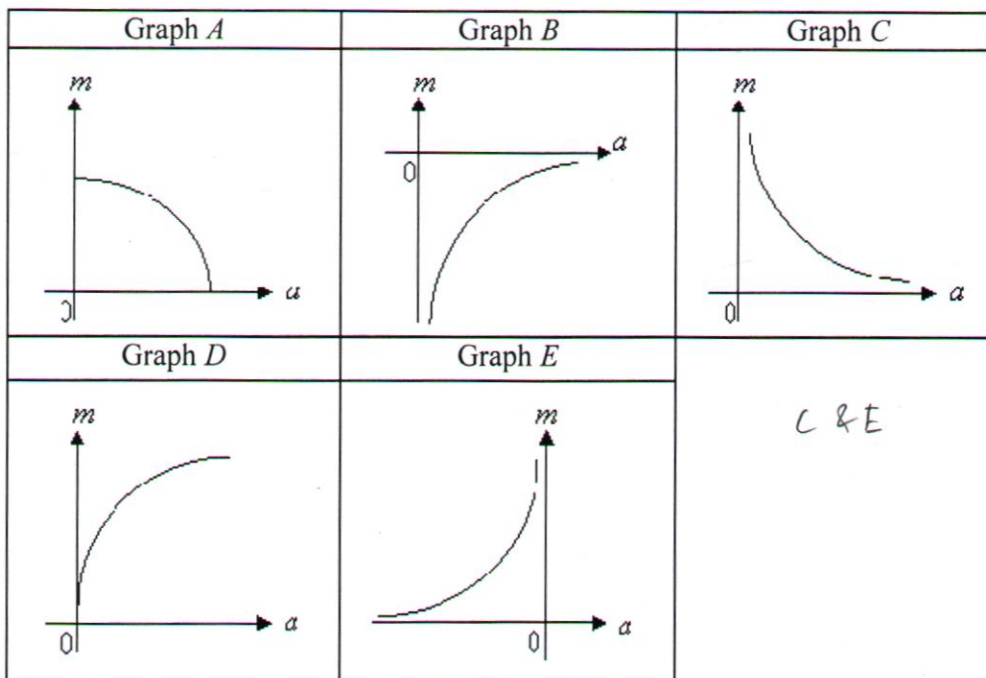


Graph (b)



Graph (c)

8. The element Andium is found in a rock on the planet Mars. The mass of the rock is given by $m = \frac{k}{a^2}$, where m is the mass of the rock, a is the number of Andium atoms the rock contains, and k is a constant. Which of the following graph(s) depict the relationship between the mass of the rock found in Mars and number of Andium atoms it contains?



Answers:

- 2ai) 60 aii) 240 c) $t = 2.75$ hr
 3bi) -1.62 bii) 0 biii) 3.125
 4b) \$400 4c) 20 items 6) $y = 2^{-x}$
 7a) $n = 1$ b) $n = 3$ c) $n = -2$
 8) C and E

Math @ Work: What do you think the graph is saying?

