West Spring Secondary School
2011 3E EM Assignment 5: Functions & Graphs

Name: ______________________ ( ) **Class:** __________ **Date:** ________

Overview

This worksheet covers the following:

1. **Graphs of Reciprocal functions**
2. **Graphs of Squared reciprocal functions**

Recap

Previously, we learned about graphs of quadratic and cubic functions.

- Linear graphs √
- Quadratic graphs √
- Cubic graphs √
- Reciprocal graphs
- Exponential graphs

You are familiar with functions in the following forms:

FUNCTION	TYPE
$y = ax^2$	Quadratic
$y = mx$	Linear
$y = c$	Constant

These are known as **power functions** and they are of the form:

$$\boxed{y = ax^n}$$

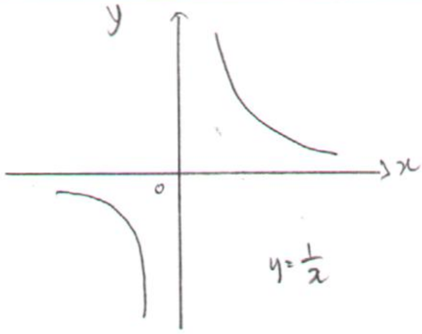
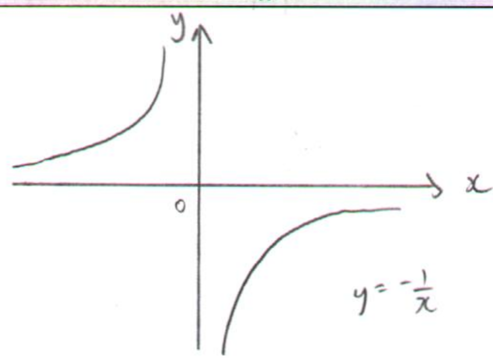
Power: rational number

Coefficient of the respective powers of the **variable x** (real number and non-zero)

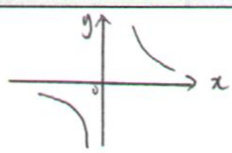
The power, n, determines the function's overall shape and behaviour.

Let's take a look at the graphs of $y = ax^n$ for $n = -2, -1, 0, 1, 2$ and 3.

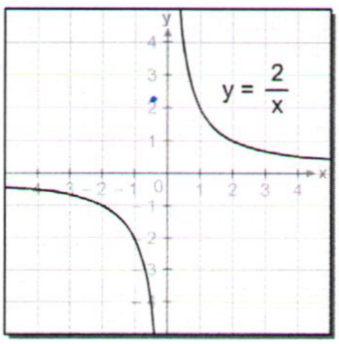
Reciprocal Graphs

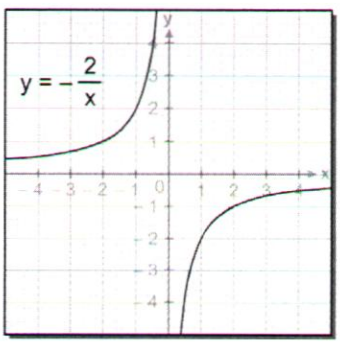
$y=\frac{1}{x}$	$y=-\frac{1}{x}$
	
Important points to note: • As x increases, y decrease • y is undefined at $x=0$	Important points to note: • Looks like $y=\frac{1}{x}$ but it is opposite • y is undefined at $x=0$

Summary

Power (n)	$y = ax^n$	Function Type	Graph	Observation
-1	$y=\frac{a}{x}$	Reciprocal or Rectangular Hyperbolic		When $x \uparrow$, $y \downarrow$ $x>0$

Let's make a comparison of the graphs $y=\frac{2}{x}$ and $y=-\frac{2}{x}$.

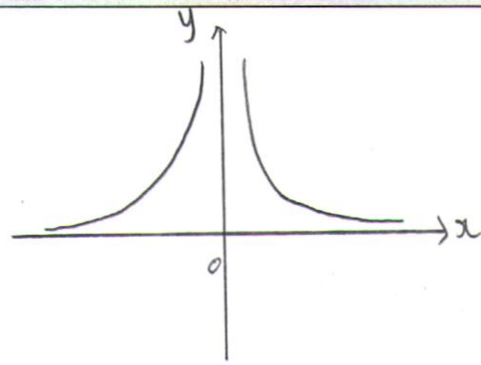
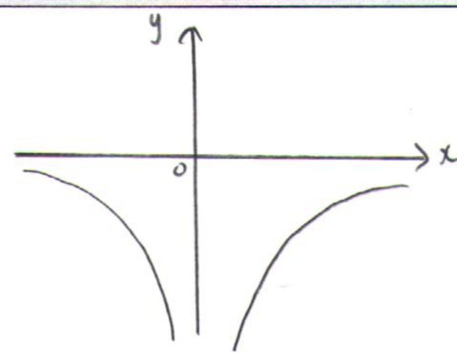




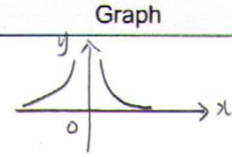
In each graph:

1. The function is undefined at $x = 0$.
2. The graph does not touch the axes.
3. The graph is not continuous and appears to have two separate parts but it must be regarded as a single graph.
4. The two parts are mirror images of each other.
5. The lines of symmetry are $y = x$ anc $y = -x$.
6. y tends to 0 as x increases or decreases infinitely.
7. x tends to 0 as y increases or decreases infinitely.

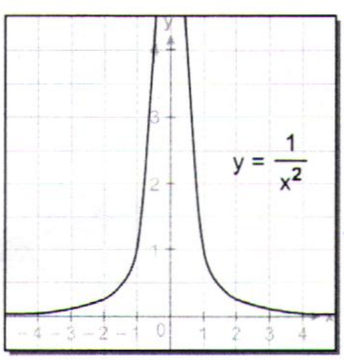
Squared-Reciprocal Graphs

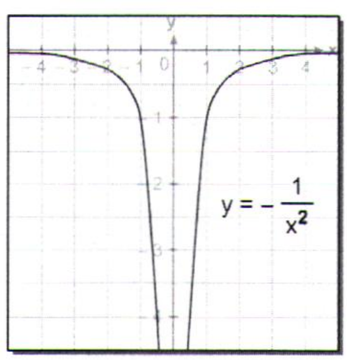
$y=\frac{1}{x^2}$	$y=-\frac{1}{x^2}$
	
Important points to note: • y is always positive	Important points to note: • Looks like $y=\frac{1}{x^2}$ but it's opposite

Summary

Power (n)	$y = ax^n$	Function Type	Graph	Observation
– 2	$y = \frac{a}{x^2}$	Square Reciprocal		It will never cut x & y axis

Let's make a comparison of the graphs $y = \frac{1}{x^2}$ and $y = -\frac{1}{x^2}$.





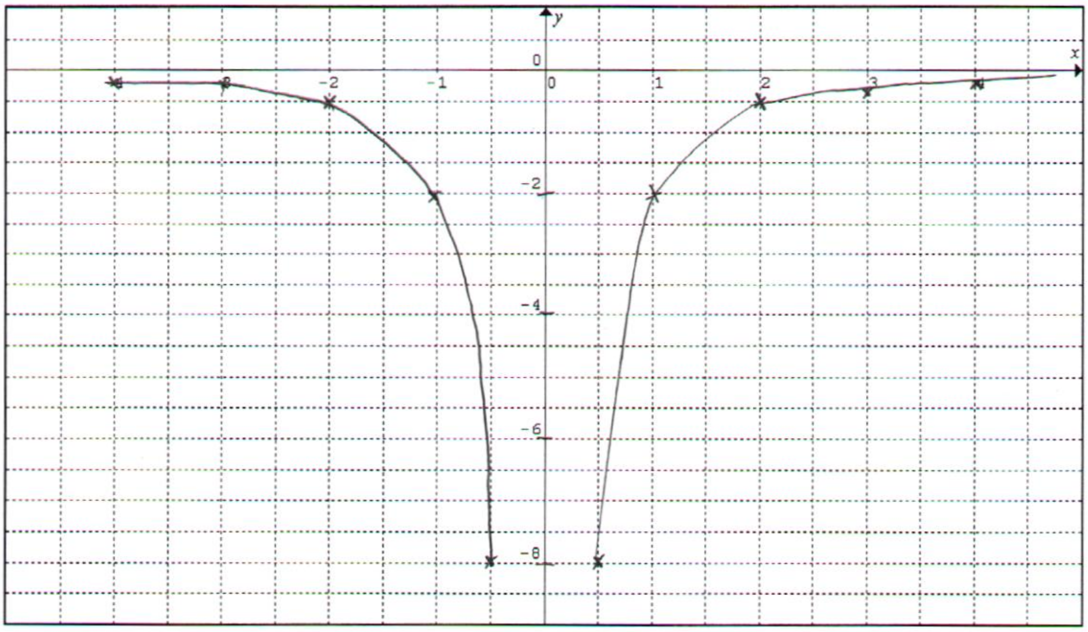
In each graph:

1. The function is undefined at $x = 0$.
2. The graph does not touch the axes.
3. The graph is not continuous and appears to have two separate parts but it must be regarded as a single graph.
4. The two parts are mirror images of each other.
5. The line of symmetry is the y-axis.
6. If a is positive:
   (a) y is always positive; so the curve lies above the axis
   (b) x tends to 0 as y increases infinitely
7. If a is negative:
   (a) y is always negative; so the curve lies below the axis
   (b) x tends to 0 as y decreases infinitely
8. y tends to 0 as x increases or decreases infinitely.

Practice

1. Complete the table below and draw the graph of $y = -\frac{2}{x^2}$ for the range $-4 \le x \le 4$

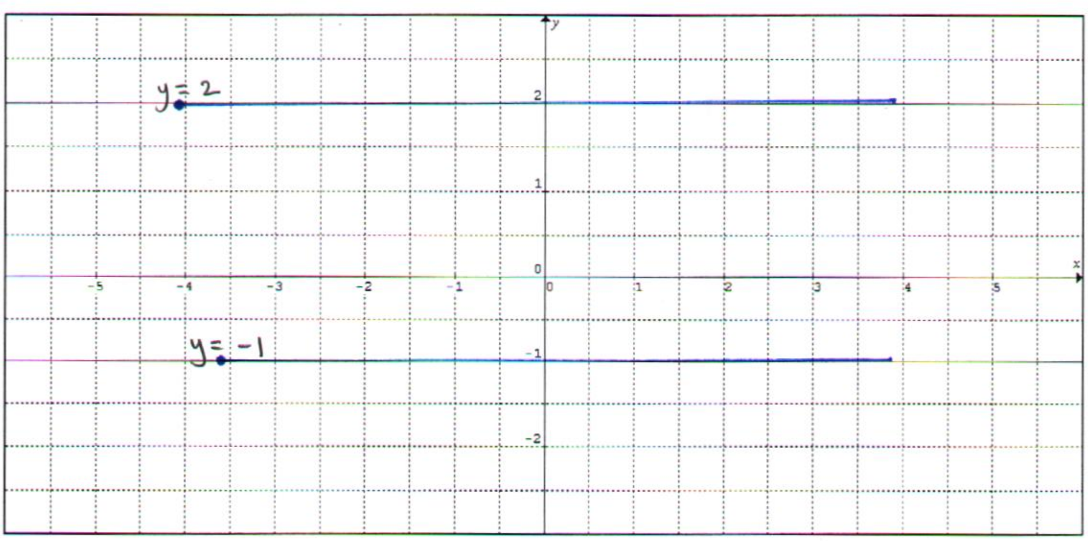
x	-4	-3	-2	-1	-0.5	0.5	1	2	3	4
y	-0.125	-0.22	-0.5	-2	-8	-8	-2	-0.5	-0.22	-0.125



Summary

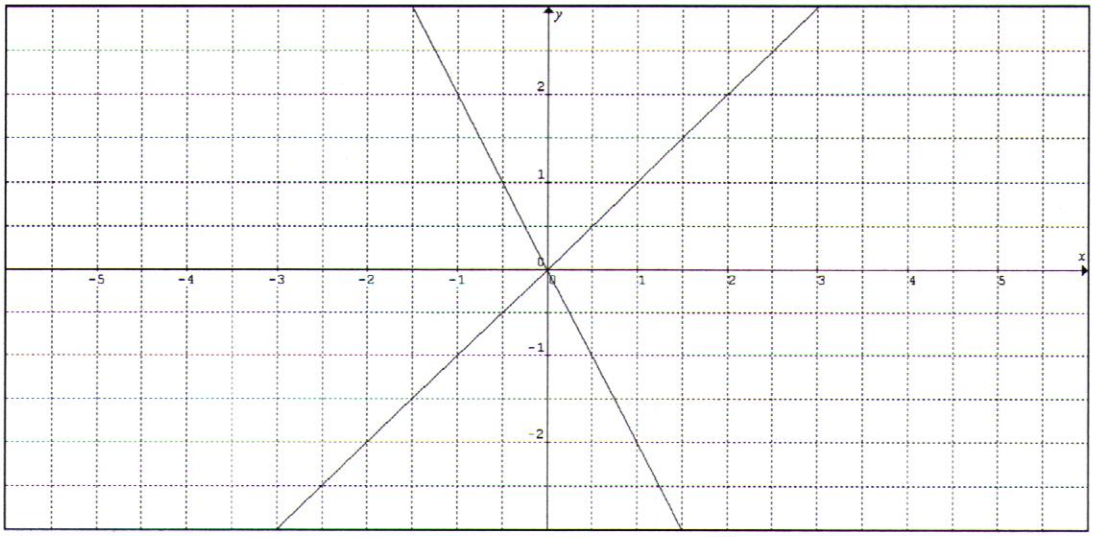
Graphs of the form $y = ax^n$ for n = 0, 1, 2, 3 and -1, -2

(A) When <u>n = 0</u>, $y = ax^0 = a$. Therefore the graph looks like:

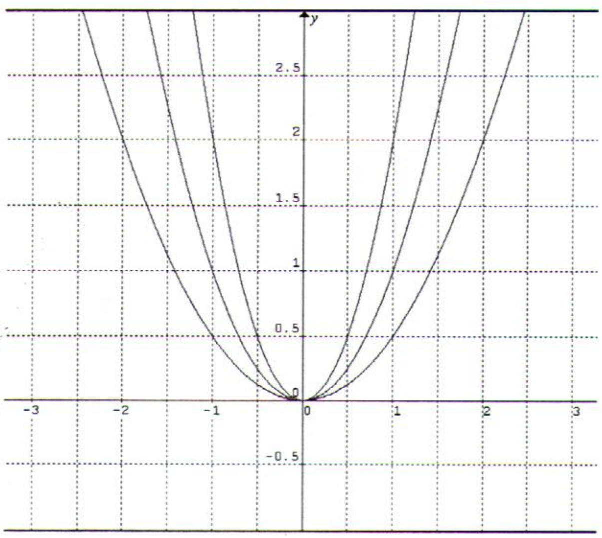


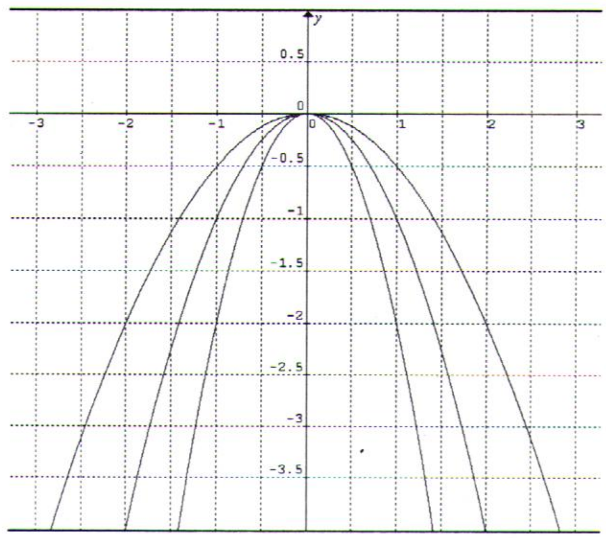
(B) When n = 1, $y = ax^1 = ax$. Therefore the graph looks like:

$y = ax^1 = ax$ is also known as linear graphs.



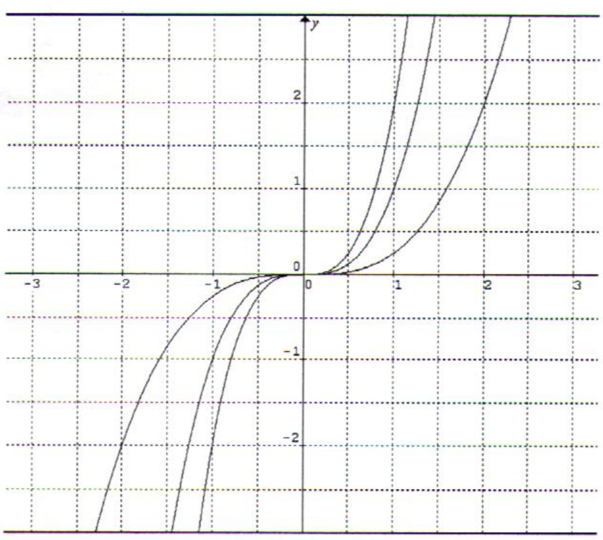
(C) When n = 2, $y = ax^2$ (quadratic graph). Therefore the graph looks like:

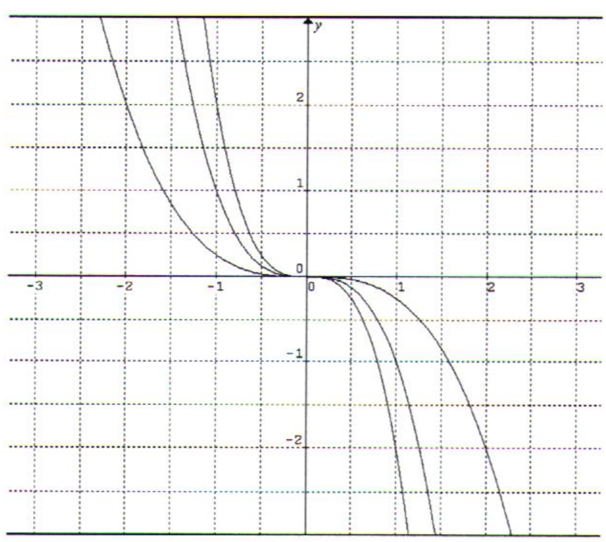




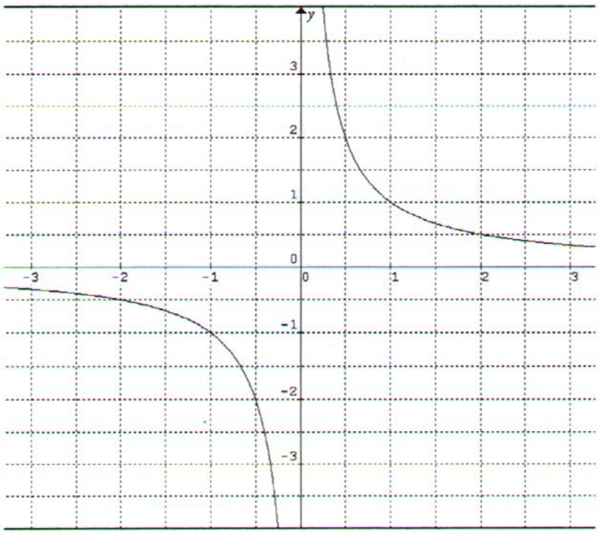
(D)

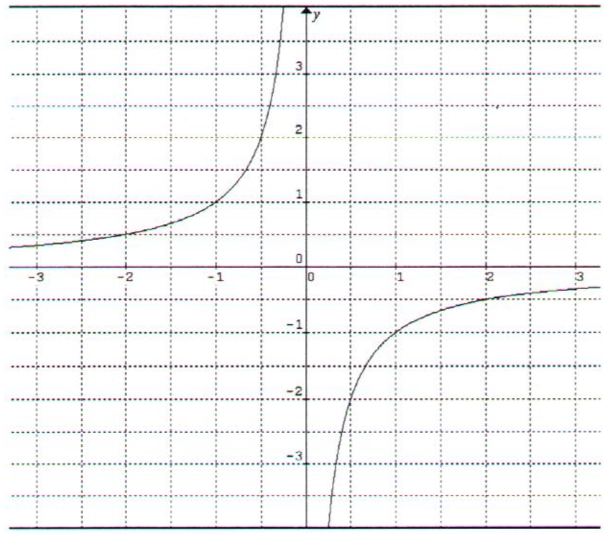
When n = 3, $y = ax^3$ (cubic graphs). Graph looks like:



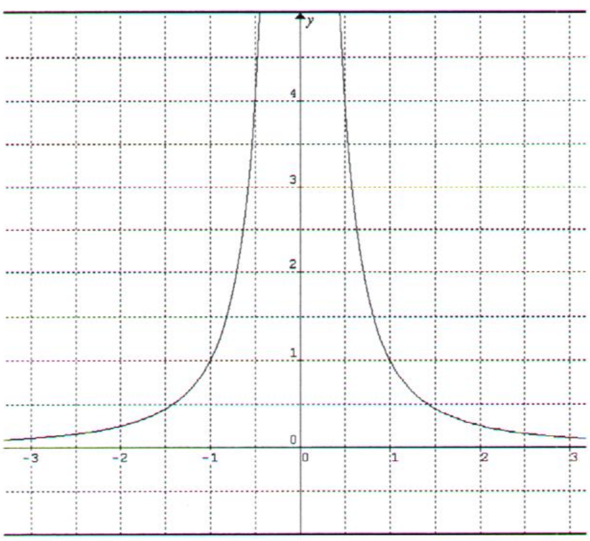


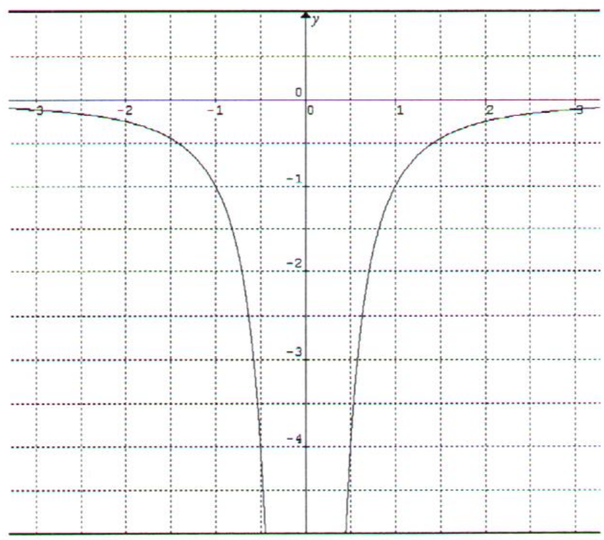
(E) When <u>n = -1</u>, $y = ax^{-1} = \frac{a}{x}$ (reciprocal graphs). Graph looks like:





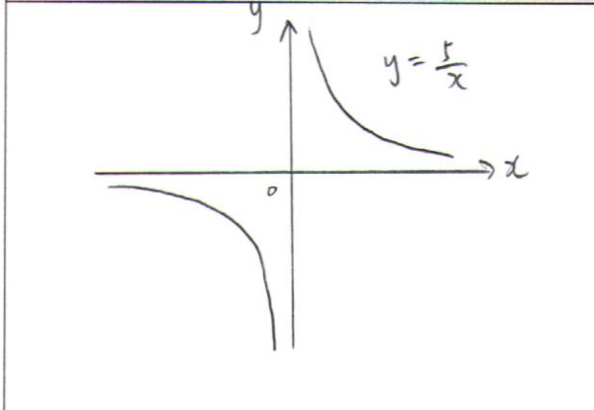
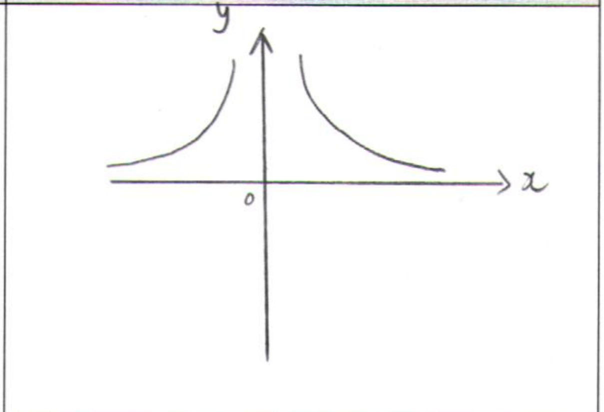
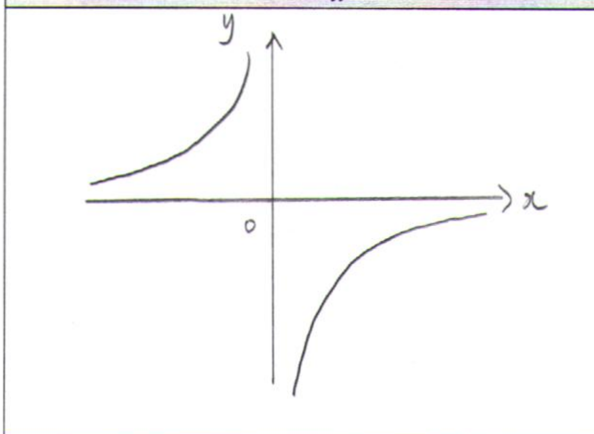
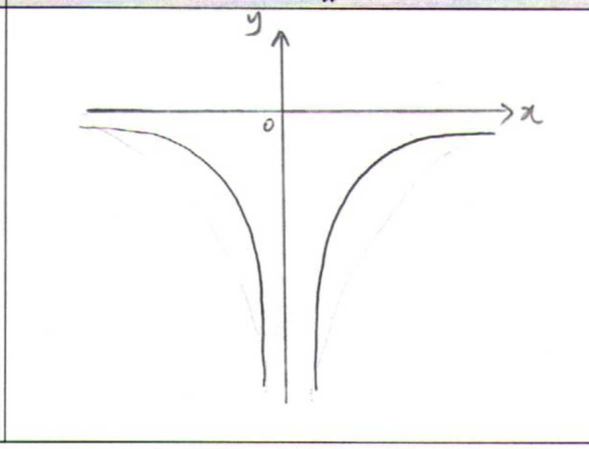
(F) When <u>n = -2</u>, $y = ax^{-2} = \frac{a}{x^2}$ (squared reciprocals). Graph looks like:





Homework

1. Sketch the following functions without refering to your textbook or notes.

$y = \frac{5}{x}$	$y = \frac{10}{x^2}$
	
$y = -\frac{1}{x}$	$y = -\frac{8}{x^2}$
	

2. The graph of $y = x^2 - 4$ is drawn.

a) Solve for $x^2 - 1 = 0$

b) Solve for $x^2 - 5 = 0$

a) $x^2 - 1 = 0$

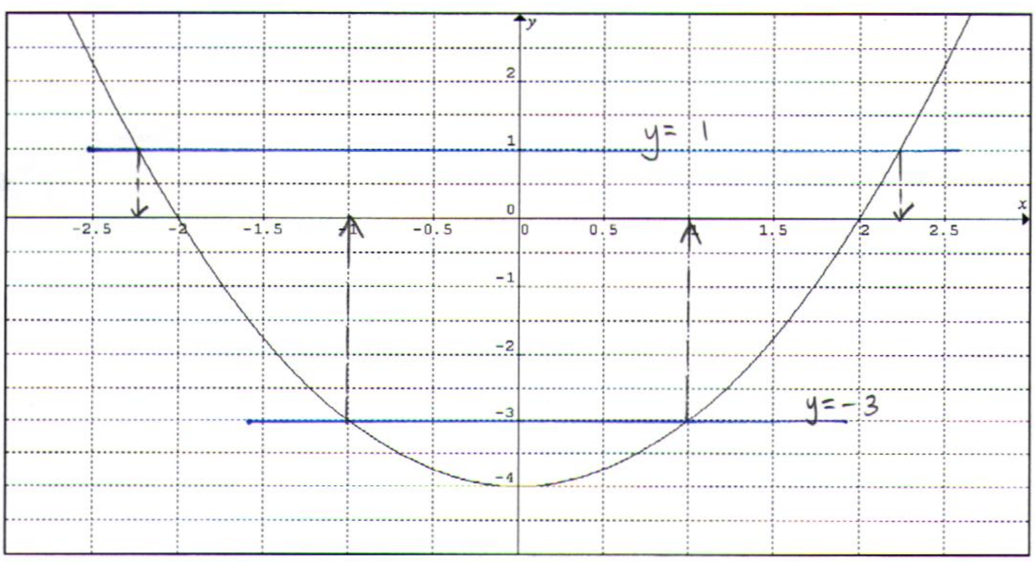
$x^2 - 1 - 3 = -3$

$x^2 - 4 = -3$

b) $x^2 - 5 = 0$

$x^2 - 5 + 1 = 1$

$x^2 - 4 = 1$



a) $x = -1$ or $x = 1$

b) $x = -2.25$ or

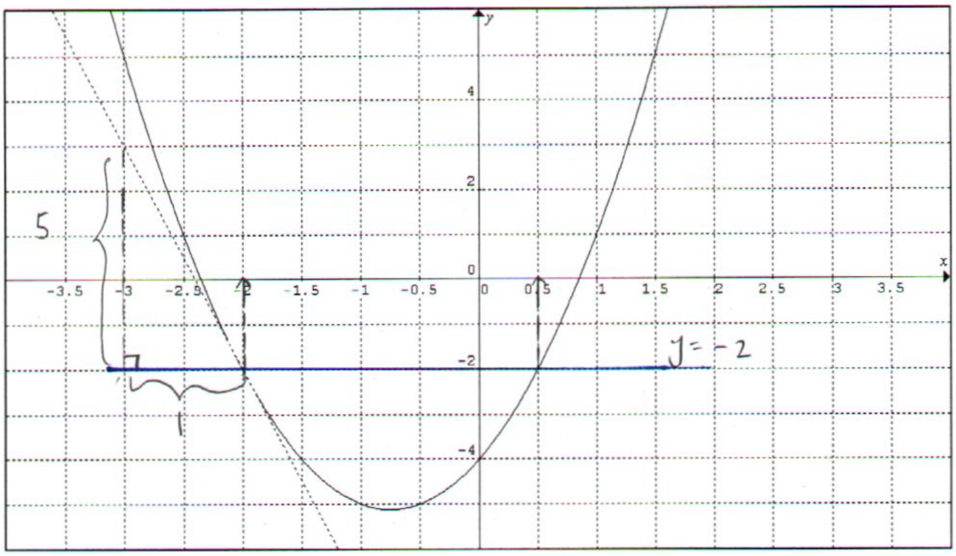
$x = 2.25$

a) $2x^2+3x-2=0$
$2x^2+3x-2-2=-2$
$2x^2+3x-4=-2$
$x=0.5$ or $x=-2$ //

3. The graph of $y=2x^2+3x-4$ is drawn.

a) Solve for $2x^2+3x-2=0$

b) Find the gradient of the graph at **x** = **-2** (Tangent is drawn already).



b) Gradient
$= -\frac{5}{1}$
$= -5$ //

4. Which of the following is the graph of

(a) $y=-5+x$, (b) $y=-\frac{4}{x}$, (c) $y=3-2x^2$?

Figure 6 Figure 2 Figure 5

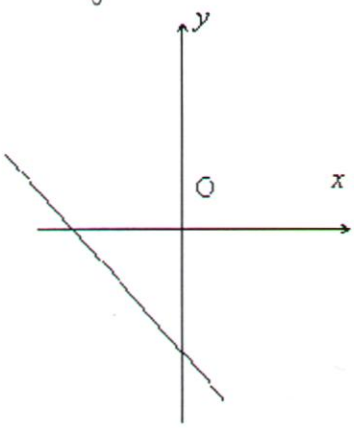


Figure 1

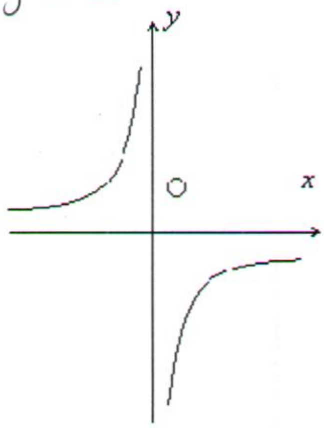


Figure 2

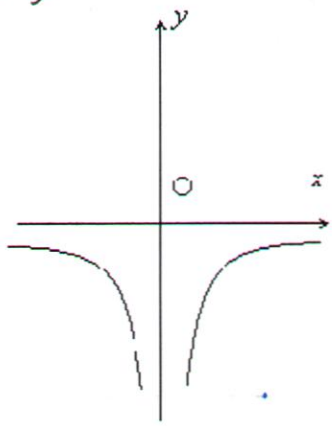


Figure 3

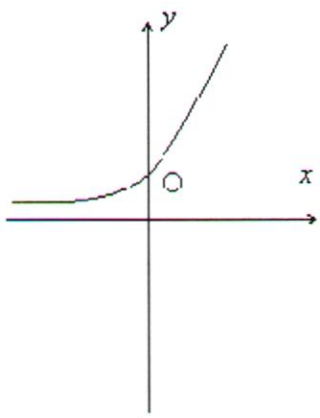


Figure 4

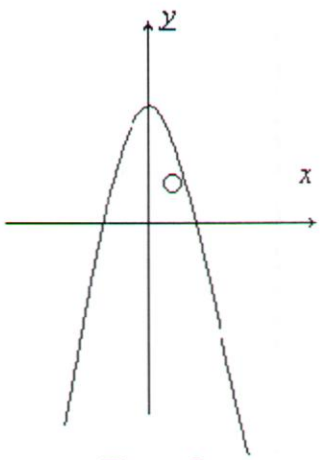


Figure 5

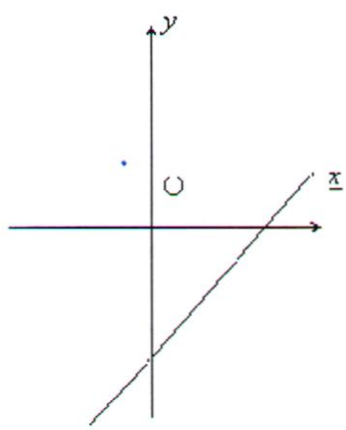


Figure 6

(a) $x^2 + 2x - 3 = 0$

$x^2 + 2x - 3 + 4 = 4$

$x^2 + 2x + 1 = 4$

$x = 1$ or $x = -3$ //

5. The graph of $y = x^2 + 2x + 1$ is drawn.

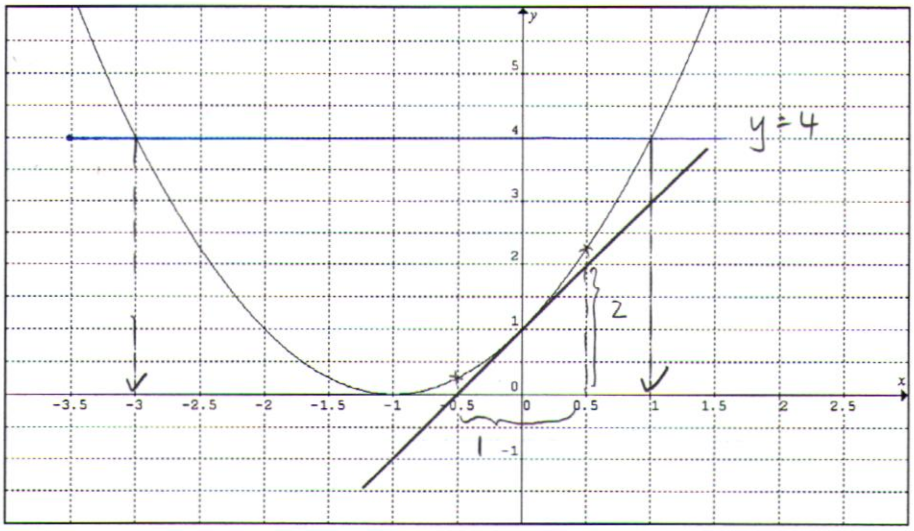
a) Solve for $x^2 + 2x - 3 = 0$

b) Find the gradient of the graph at *(0, 1)* by drawing a suitable tangent.

b) Gradient

$= \frac{2}{1}$

$= 2$ //



(a) $-x^2 + 2x + 3 = 0$

$-x^2 + 2x + 3 - 2 = -2$

$-x^2 + 2x + 1 = -2$

$x = 3$ or $x = -1$ //

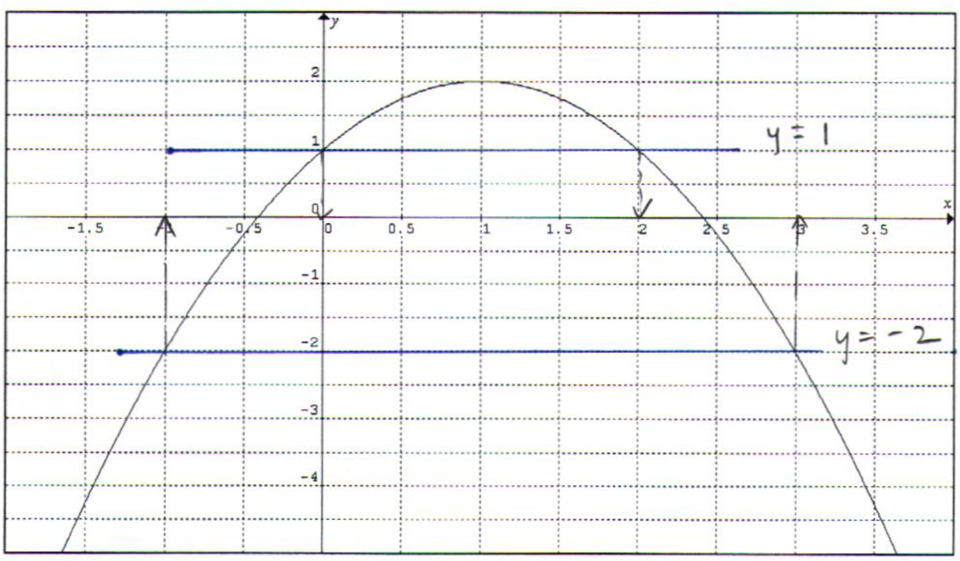
6. The graph of $y = -x^2 + 2x + 1$ is drawn.

a) Solve for $-x^2 + 2x + 3 = 0$

b) Solve for $-x^2 + 2x = 0$

b) $-x^2 + 2x + 1 = 1$

$x = 0$ or $x = 2$ //



(a) $-x^3 + 2x + 0.5 = 0$

$-x^3 + 2x + 0.5 + 0.5 = 0.5$

$-x^3 + 2x + 1 = 0.5$

$x = -1.25$ or $x = -0.25$ or $x = 1.5$ //

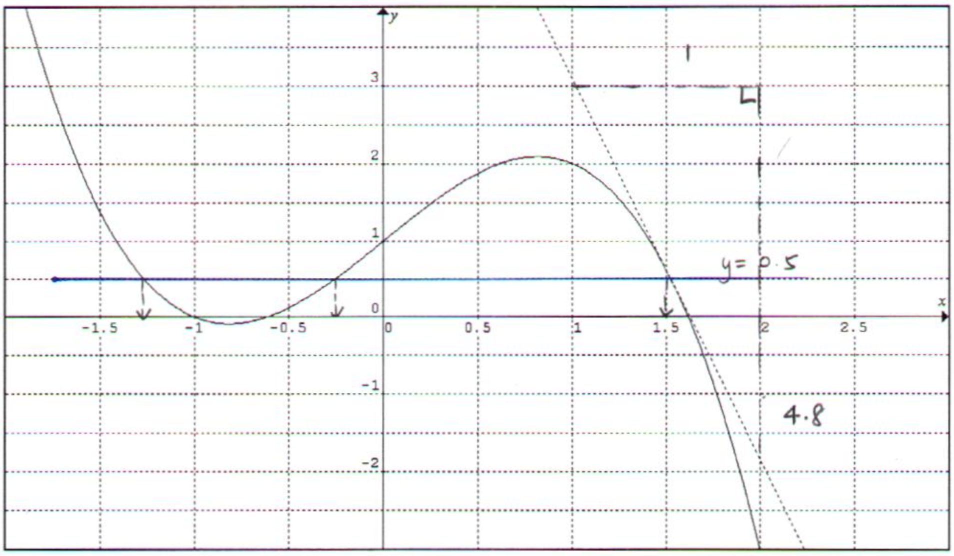
7. The graph of $y = -x^3 + 2x + 1$ is drawn.

a) Solve for $-x^3 + 2x = -0.5$

b) Find the gradient of the graph at ***x* = *1.5*** (Tangent is drawn already).

b) Gradient $= -\frac{4.8}{1}$

$= -4.8$ //



8. The graph of $y = 2x^3 + 2x^2 + x$ is drawn.

a) Solve for $2x^3 + 2x^2 + x = 1$

b) Solve for $2x^3 + 2x^2 + 2x = 0$

a) $2x^3 + 2x^2 + x = 1$

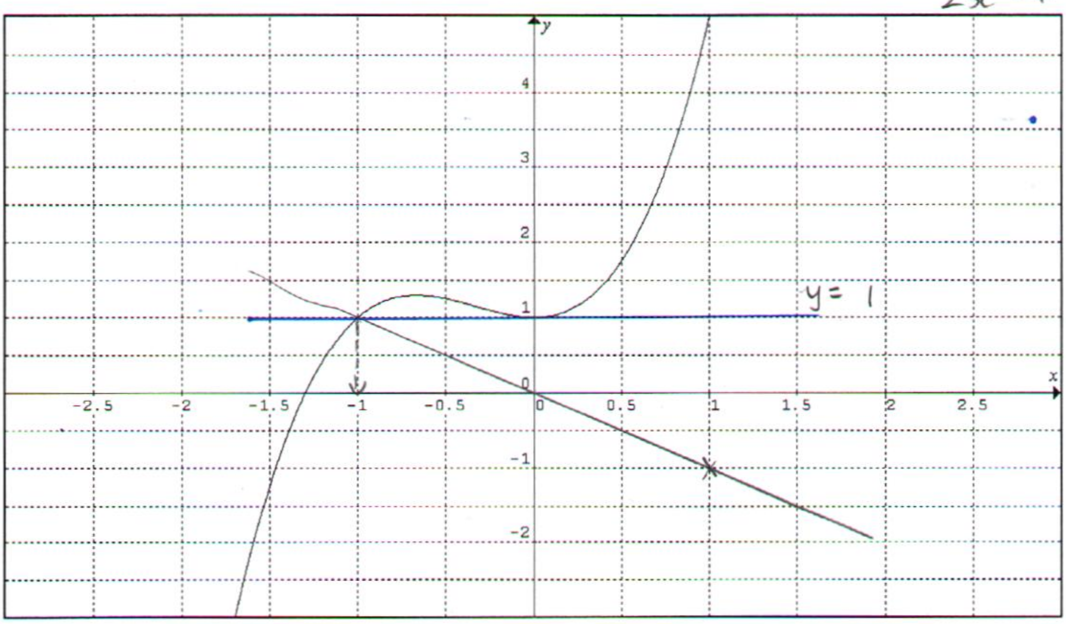
$x = -1$ or $x = 0$

b) $2x^3 + 2x^2 + 2x = 0$

$2x^3 + 2x^2 + 2x - x = -x$

$2x^3 + 2x^2 + x = -x$

$x = -1$ //



Answers:

2a) x=-1, 1	2b) x=2.25, -2.25	3a) x=0.5, -2	3b) -5	4) 6, 2, 5	5a) x=1, -3	5b) 1.8
6a) x=3, -1	6b) x=0, 2	7a) x=-1.25, -0.25, 1.5		7b) -4.8	8a) x=-1, 0	8b) x=-1