West Spring Secondary School
2011 3E EM Assignment 4: Functions & Graphs

Name: ____________________ (   ) Class: __________ Date: ________

Overview

This worksheet covers the following:

1. **Application of graphs of quadratic equation**
2. **Graphs of Cubic functions**

Introduction

Previously we learned we can express quadratic functions in the form of :

- $y=(x-h)^2+k$
- $y=-(x-h)^2+k$
- $y=(x-p)(x-q)$
- $y=-(x-p)(x-q)$

From the graph, we can show:

- min or max point (how do we know?)
- line of symmetry
- y-intercept
- x-intercept (if applicable)

Now that we have learned how to sketch quadratic functions, let's see how we can apply this knowledge to real-world problems.

Example 1:

The weekly profit *y* (in $) for producing and selling *x* vases is given by $y=-x^2+60x$

a) Sketch the graph for $x \geq 0$

b) What is the maximum weekly profit?

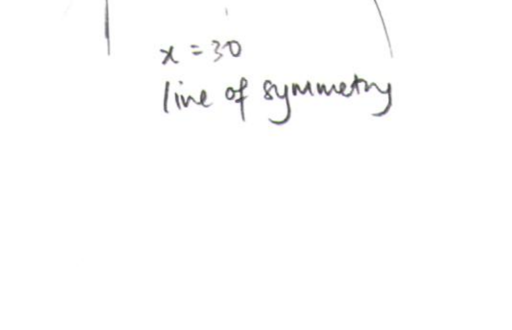
c) How many vases must be produced and sold in order to obtain this maximum profit?

(Ans: b) $900 c) 30 vases)

(a) $y = -x^2+60x$

$= -(x^2-60x)$

$y = -x(x-60)$



(b) Maximum weekly profit

= $900

(c) No. of vases = 30

Practice 1:

The daily profit y (in dollars) from the sales of a certain product can be represented by the function $y = 80x - x^2$, where x is the number of products produced in a day.

a) Sketch the graph for $x \geq 0$.

b) How many products must be produced daily in order to maximise the profit? What is the maximum daily profit?

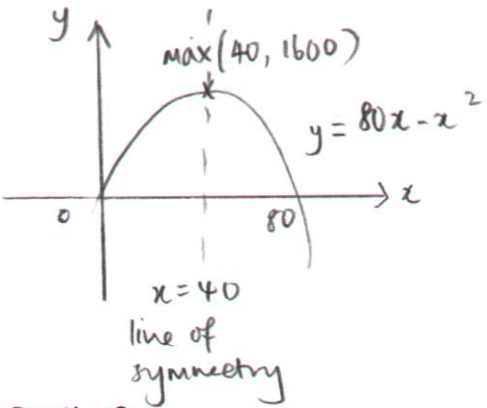
(Ans: b) 40 products)

(b) No. of products = 40

Maximum daily profits = \$ 1600

(a) $y = 80x - x^2$

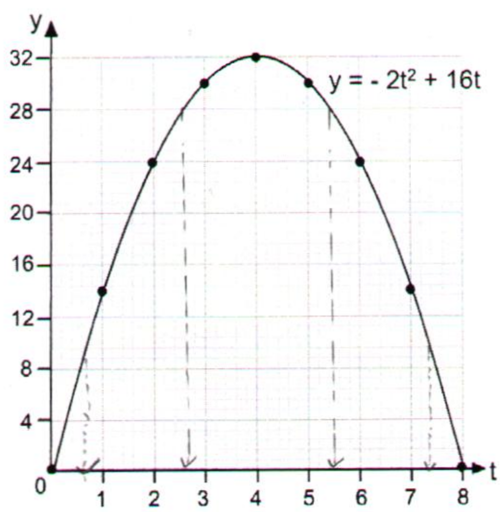
$= x(80 - x)$



Practice 2:

When a ball is thrown into the air, its height after t seconds is given by $y = -2t^2 + 16t$

t	0	1	2	3	4	5	6	7	8
$y = -2t^2 + 16t$	0	14	24	30	32	30	24	14	0



a) When is the ball at ground level (zero height)?

b) At which times is the ball 10m above the ground?

c) At which times is the ball 28m above the ground?

a) t = 8 s

b) t = 0.6 s or t = 7.4 s

c) t = 2.6 s or t = 5.4 s

In addition to quadratic functions, we also need to know how the following types of functions look like:

- **Linear graphs**
- **Quadratic graphs**
- **Cubic graphs**
- **Reciprocal / Squared-Reciprocal graphs**
- **Exponential graphs**

The function $y = ax^n$, where a is a real constant and n is a rational number, is called a **power function**. We shall study the graph of this function for $n = -2, -1, 0, 1, 2$ and 3, in this section.

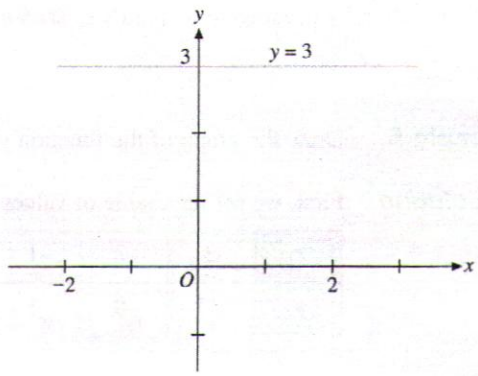
A. Graph of $y = ax^n$ for $n = 0, 1, 2$, and 3

<u>Case $n = 0$</u>

When $n = 0$, $y = ax^n = ax^0$.
Since $x^0 = 1$ for all x except $x = 0$, we shall regard the function as

$$y = a.$$

This is the **constant function** that we learnt in Secondary Two. The diagram on the right shows the graph of two constant functions, $y = 3$ (i.e. $a = 3$) and $y = -2$ (i.e. $a = -2$).

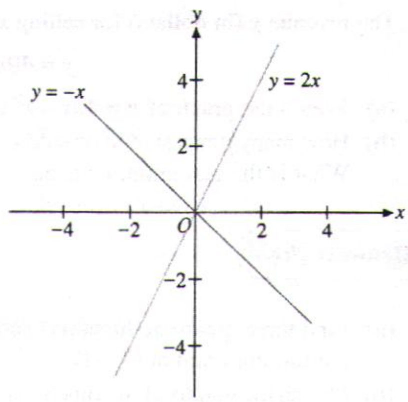


<u>Case $n = 1$</u>

When $n = 1$, the function $y = ax^n$ becomes

$$y = ax.$$

We learnt that this is a linear function. The diagram on the right shows the graphs of $y = 2x$ and $y = -x$.

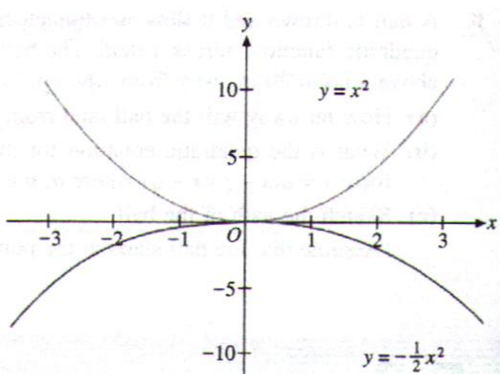


<u>Case $n = 2$</u>

When $n = 2$, $y = ax^n$ becomes

$$y = ax^2.$$

We know that this is a quadratic function. The diagram on the right shows the graphs of $y = x^2$ and $y = -\frac{1}{2}x^2$.



Case $n = 3$

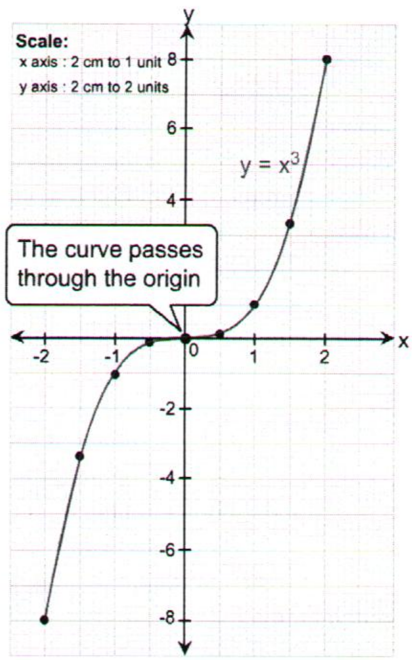
When $n = 3$, $y = ax^n$ becomes

$$y = ax^3.$$

This function is new to us. It is known as a **cubic function**. Let us draw the graph of this function by plotting some points as shown in the following example.

Example 1:

Let's look at the graph of $y = x^3$



We first set up a table of values of x from -2 to +2

x	-2	-1.5	-1	-0.5	0	0.5	1	1.5	2
y	-8	-3.375	-1	-0.125	0	0.125	1	3.375	8

Choose a suitable scale for the axes to fit in these values of x and y

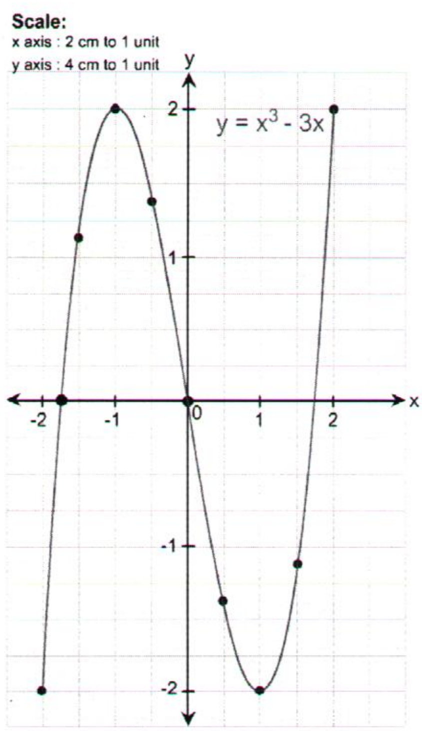
Here we choose the scale :
2 cm to represent 1 unit on the x axis
2 cm to represent 2 units on the y axis

Let us plot few more points to get a better idea of the shape of the curve

Join these points with a smooth curve.

Question: Does the graph have rotational symmetry?
If yes, what's the order?

Now let's draw the graph of $y = x^3 - 3x$ for values of x from -2 to +2 taking intervals of 0.5.



We first set up a table of values of x from -2 to +2

x	-2	-1.5	-1	-0.5	0	0.5	1	1.5	2
y	-2	1.125	2	1.375	0	-1.375	-2	-1.125	2

Join these points with a smooth curve.

This curve has rotational symmetry of order 2 about the origin

Question:

Using the graph, solve the equation $x^3 - 3x = 0$

How many solutions are there?

Practice

Draw the graph of $y = x^3 - 6x + 1$ for values of x from -3 to +3.
The table of values is given below.

x	-3	-2.5	-2	-1.5	-1	-0.5	0	0.5	1	1.5	2	2.5	3
y	-8	0.375	5	6.625	6	3.875	1	-1.875	-4	-4.625	-3	1.625	10

Use your graph to solve the following:

a) $x^3 - 6x + 1 = 0$

b) $x^3 - 8x = 0$

c) Find the range of x such that $x^3 - 6x + 1 > 2x + 1$

(Ans: a) -2.55, 0.15, 2.4 b) -2.9, 0, 2.8 c) -2.9<x<0 and x>2.8)

(b) $x^3 - 8x = 0$

$x^2 - 8x + 2x + 1 = 2x + 1$

$x^2 - 6x + 1 = 2x + 1$

Homework

1. **Answer this question neatly on a piece of graph paper.**
The table below gives the x and y coordinates of points which lie on the curve
$y = 8x^3 + 2x^2 - 13x + 3$

x	-2	-1.5	-1	-0.5	0	0.5	1	1.5
y	-27	0	10	9	3	-2	0	15

a) Taking a suitable scale, draw the graph.

b) Use your graph to solve the following equation

i) $8x^3 + 2x^2 - 13x + 3 = 0$

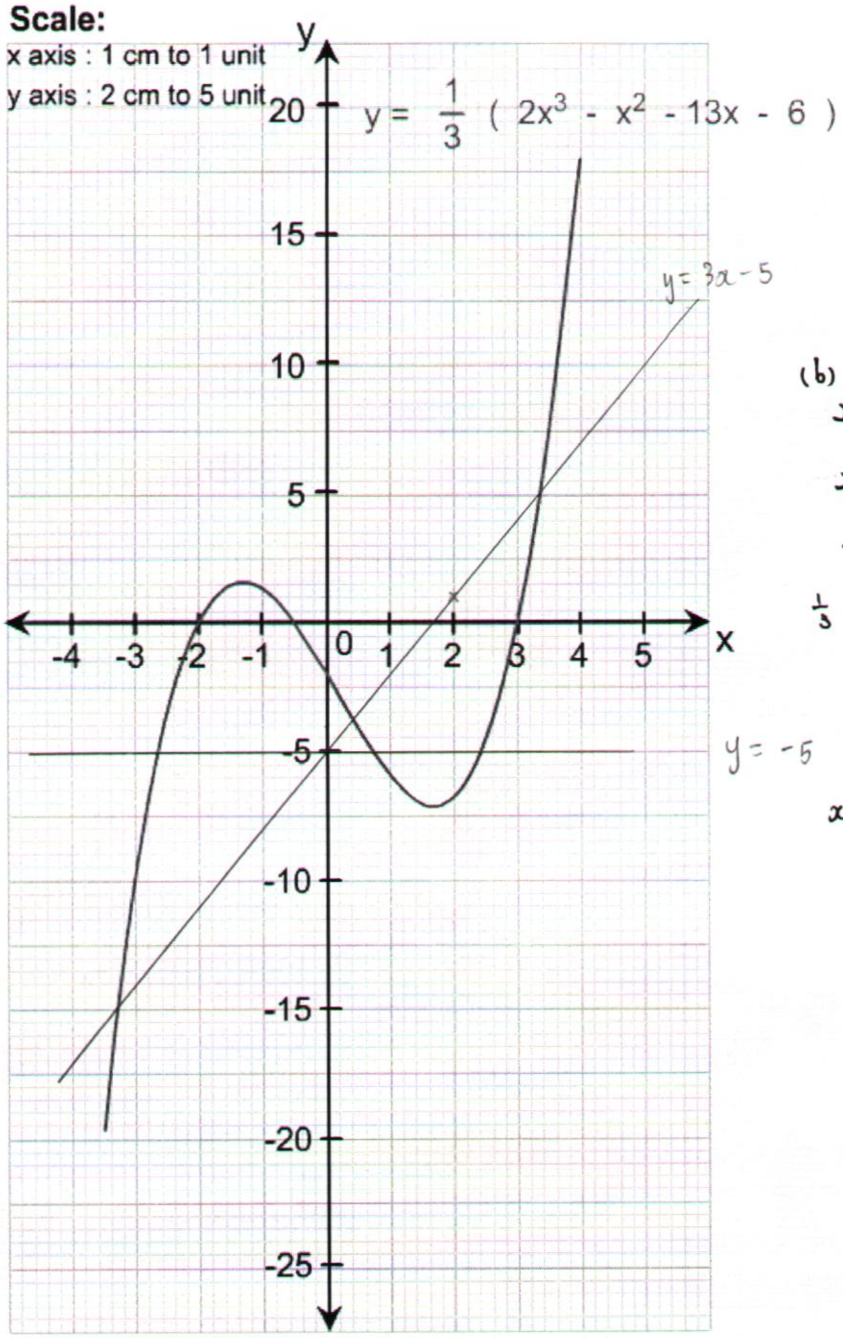
ii) $8x^3 + 2x^2 - 13x - 2 = 0$

iii) $8x^3 + 2x^2 - 19x + 1 = 0$

2. The graph of the curve $y = \frac{1}{3}(2x^3 - x^2 - 13x - 6)$ is given.
By plotting a suitable straight line on the graph, solve the following equations:

(a) $2x^3 - x^2 - 13x + 9 = 0$

(b) $2x^3 - x^2 - 22x + 9 = 0$



(a)

$2x^3 - x^2 - 13x + 9 = 0$

$2x^3 - x^2 - 13x + 9 - 15 = -15$

$2x^3 - x^2 - 13x - 6 = -15$

$\frac{1}{3}(2x^3 - x^2 - 13x - 6) = \frac{1}{3}(-15)$

$\frac{1}{3}(2x^3 - x^2 - 13x - 6) = -5$

$x = -2.7$ or $x = 0.8$ or $x = 2.4$

(b)

$2x^3 - x^2 - 22x + 9 = 0$

$2x^3 - x^2 - 22x + 9x + 9 - 15 = 9x - 15$

$2x^3 - x^2 - 13x - 6 = 9x - 15$

$\frac{1}{3}(2x^3 - x^2 - 13x - 6) = \frac{1}{3}(9x - 15)$

$\frac{1}{3}(2x^3 - x^2 - 13x - 6) = 3x - 5$

$x = -3.3$ or $x = 0.45$ or $x = 3.4$

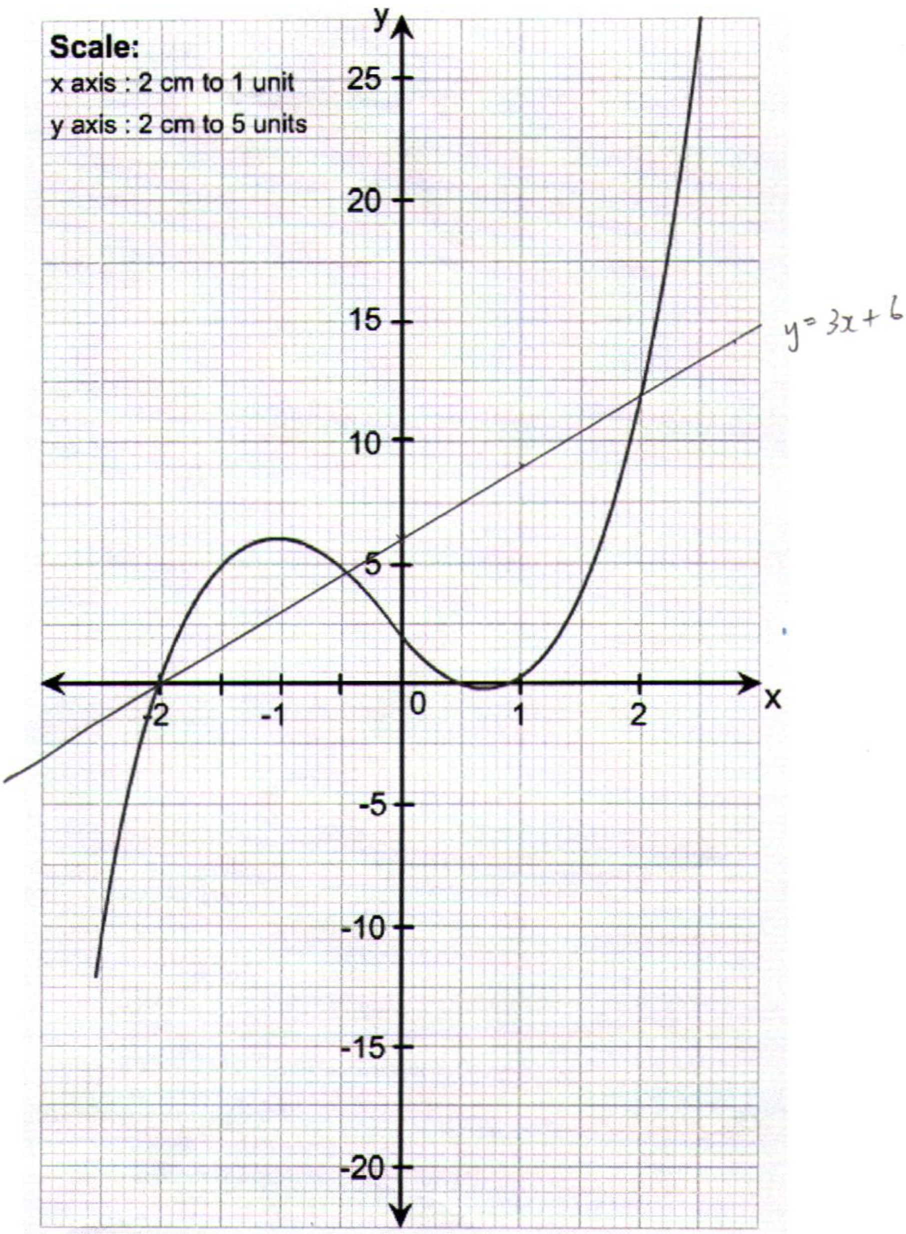
3. Given the graph of $y = 2x^3 + x^2 - 5x + 2$, add a line to the graph in order to solve $2x^3 + x^2 - 8x - 4 = 0$

$2x^3 + x^2 - 8x - 4 = 0$

$2x^3 + x^2 - 8x + 3x - 4 + 6 = 3x + 6$

$2x^3 + x^2 - 5x + 2 = 3x + 6$

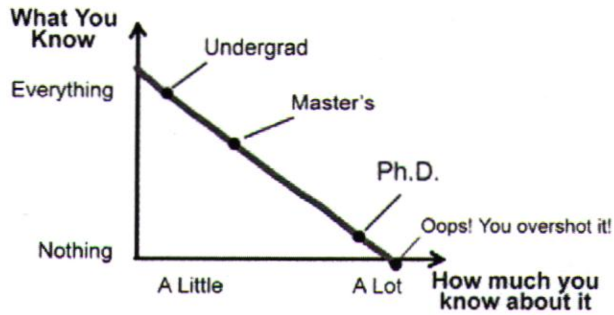
$x = -2$ or $x = -0.45$ or $x = 2$



Deadline: Apr 2011

Answers:
2a) x = -2.7, 0.6, 2.4 b) x = -3.4, 0.4, 3.25
3) x = 2, -0.45, -2

What You Know vs How much you know about it



My Level of Understanding (0-100%):

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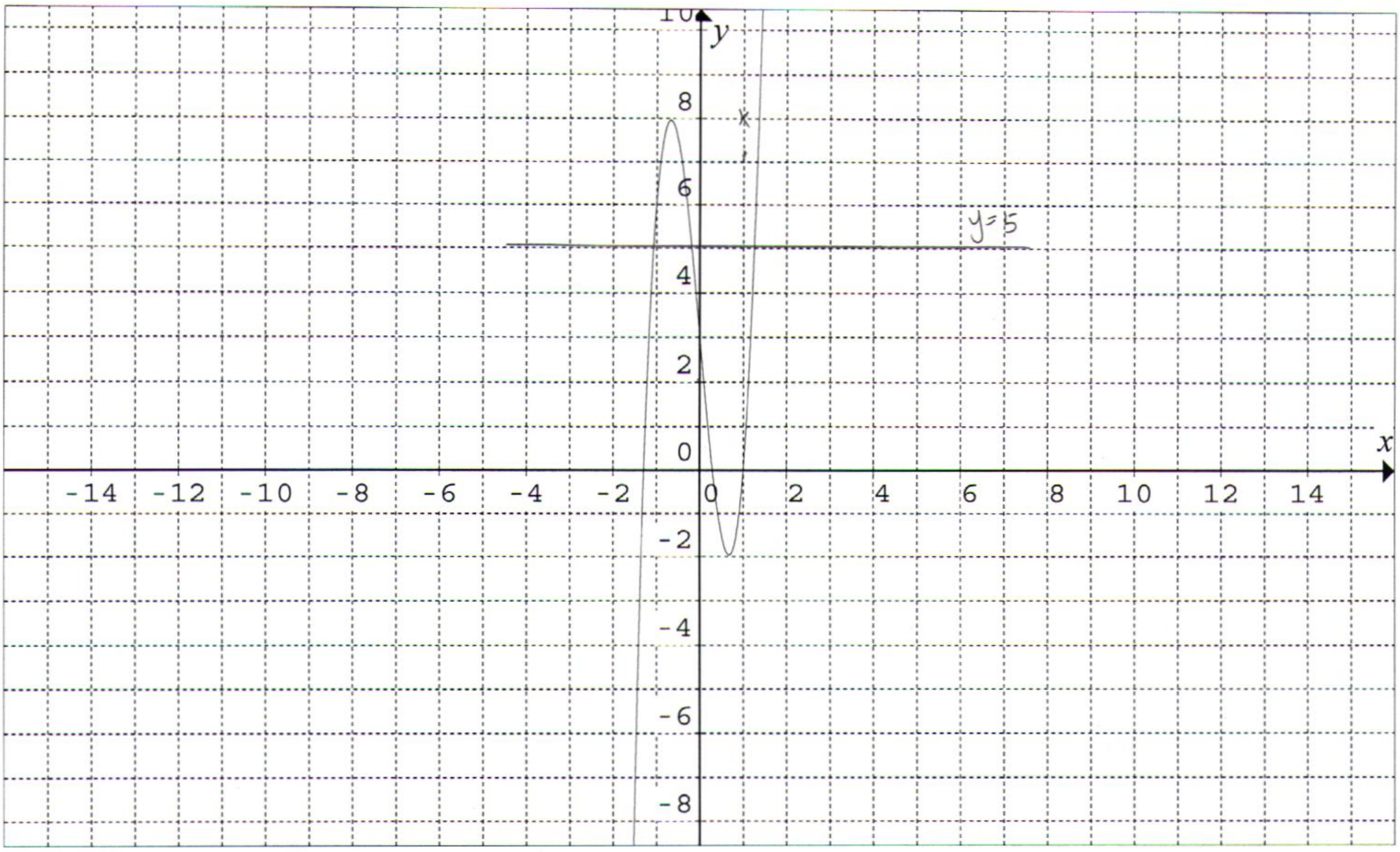

Equations on screen:

1. y=8x^3+2x-13x+3

(ii) $8x^3 + 2x^2 - 13x - 2 + 5 = 5$

$8x^3 + 2x^2 - 13x + 3 = 5$

(iii) $8x^3 + 2x^2 - 19x + 1 = 0$

$8x^3 + 2x^2 - 19x + 6x + 1 + 2 = 6x + 2$

$8x^3 + 2x^2 - 13x + 3 = 6x + 2$

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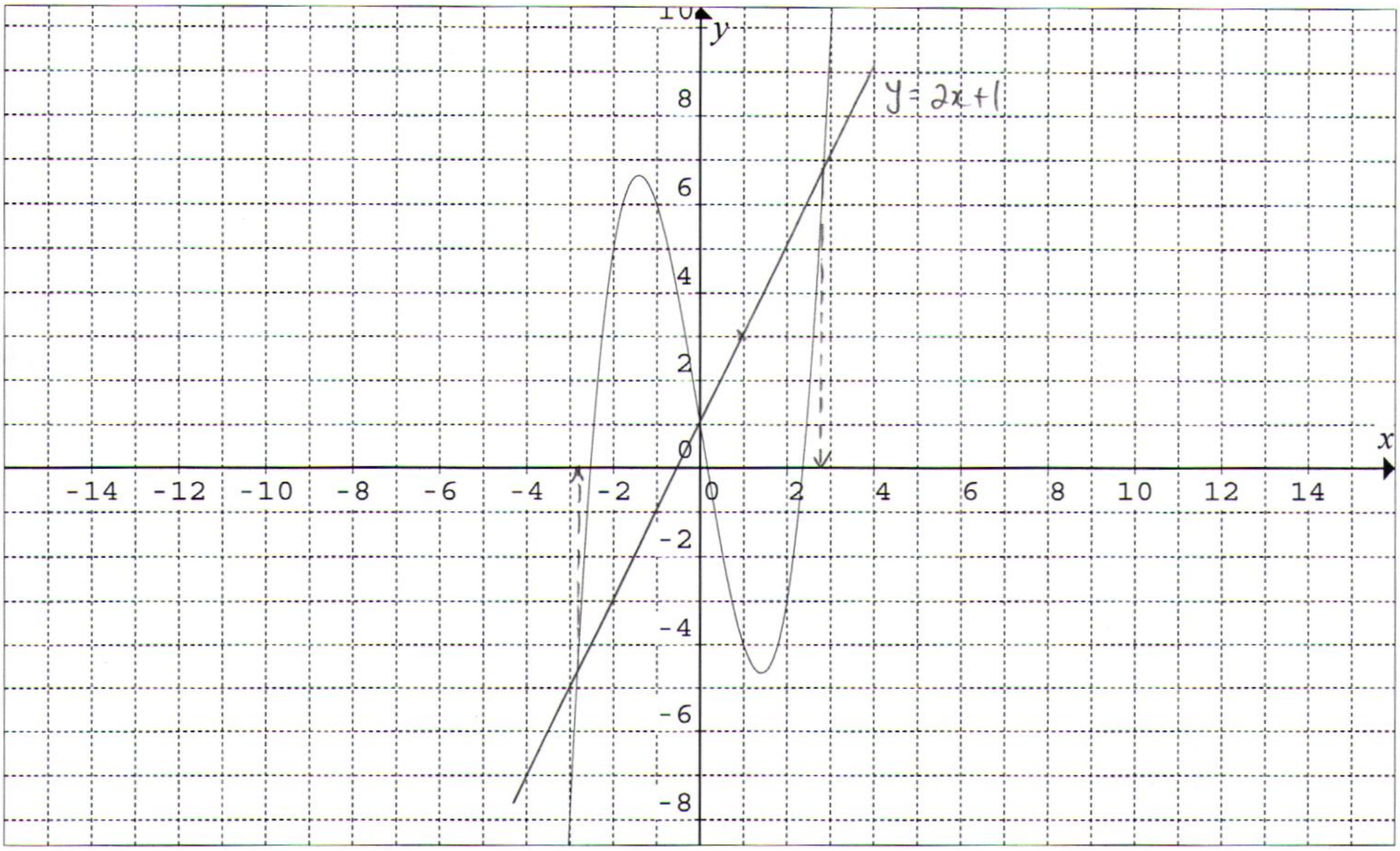

Equations on screen:

1. y=x^3-6x+1