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West Spring Secondary School
2011 3E EM Assignment 3: Functions & Graphs

Name: _____ ( ) **Class:** _____ **Date:** _____

Overview

This worksheet covers the following:

1. Recognizing shape of quadratic functions
2. Sketching quadratic functions of the form $y = (x-p)(x-q)$ **or** $y = -(x-p)(x-q)$

Introduction

Previously we learned we can express quadratic functions in the form of $y = (x-h)^2 + k$ **or** $y = -(x-h)^2 + k$, then sketch the graph.

From the graph, we need to show:

- min or max point (how do we know?)
- line of symmetry (state the equation)
- y-intercept

Today, we learn another form in which quadratic functions can be expressed.
They are:

$y = (x-p)(x-q)$ **or** $y = -(x-p)(x-q)$

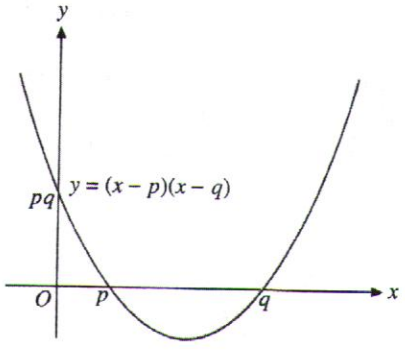
How do we sketch these functions?

Let's take a closer look...

$$y = (x-p)(x-q)$$

Consider $y = (x-p)(x-q)$ where p and q are constants.

Graph of $y = (x-p)(x-q)$

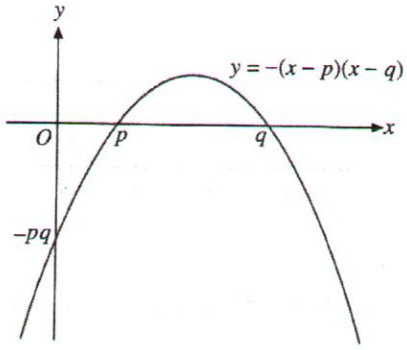


Important points:

- shape of graph
- label where it **cuts the x-axis at *x* = *p* and *x* = *q***
- label where it cuts the y-axis
- line of symmetry at $x = \frac{p+q}{2}$
- the max/min points occurs at when $x = \frac{p+q}{2}$

Similarly, we can sketch the graph of $y = -(x-p)(x-q)$.

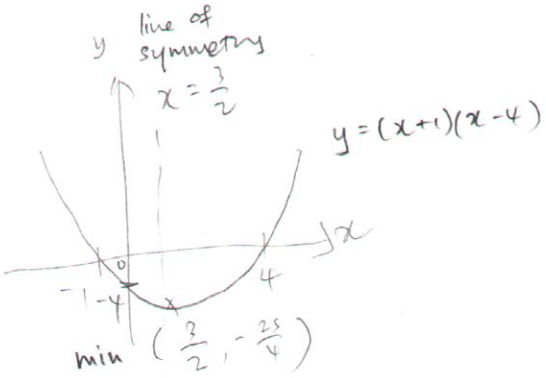
Graph of $y = -(x-p)(x-q)$



Important points:

- shape of graph
- label where it **cuts the x-axis at x = p and x = q**
- label where it cuts the y-axis
- line of symmetry at $x = \dfrac{p+q}{2}$
- the max/min points occurs at when $x = \dfrac{p+q}{2}$

$x = -1$ or $x = 4$



Example:

1. Sketch the graph of $y = (x+1)(x-4)$. Find:
   - the minimum or maximum point of the function
   - the line of symmetry (state its equation)
   - the x and y intercepts

min $x = \dfrac{-1+4}{2}$

$= \dfrac{3}{2}$

when $x = \dfrac{3}{2}$,

$y = -\dfrac{25}{4}$

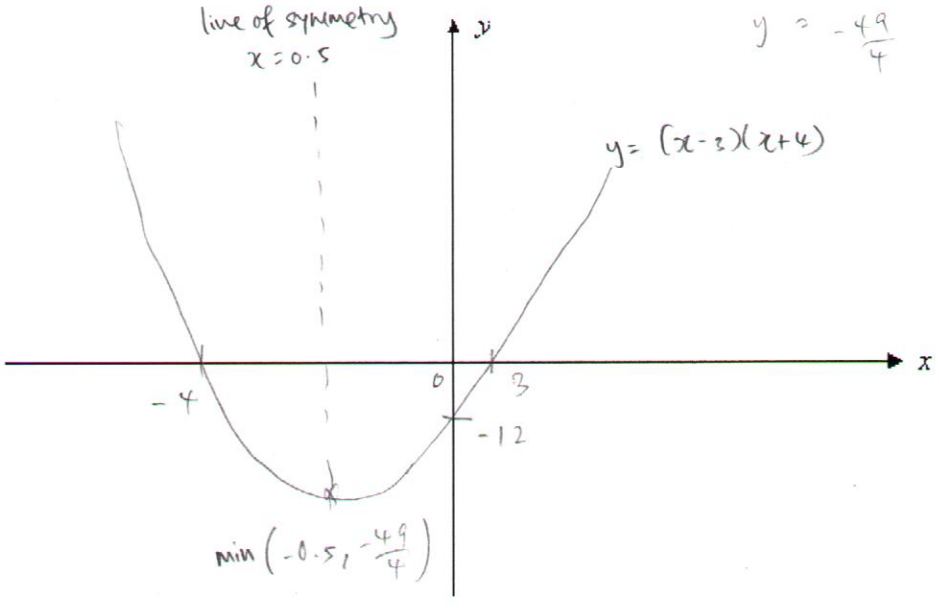
∴ min point $= \left(\dfrac{3}{2}, -\dfrac{25}{4}\right)$

$x = 3$ or -4

2. Sketch $y = (x-3)(x+4)$ in the axes provided below, showing clearly:
   - the minimum or maximum point of the function
   - the line of symmetry (state its equation)
   - the x and y intercepts

min $x = -0.5$

$y = -\dfrac{49}{4}$



Practice

1. Sketch the graphs for the following quadratic equations, showing clearly:
   - x-intercept
   - y-intercept
   - line of symmetry (state the equation)
   - max/min point (state the coordinates)

a) $y=(x-1)(x-2)$

$x=1$ or 2

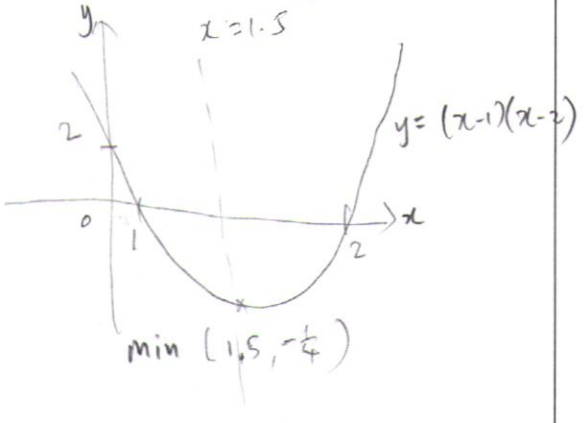
$x=\frac{1+2}{2}$

$=1.5$

$y=-\frac{1}{4}$

min $\left(1.5,-\frac{1}{4}\right)$

line of symmetry $x=1.5$



b) $y=(x+3)(x-2)$

$x=-3$ or 2

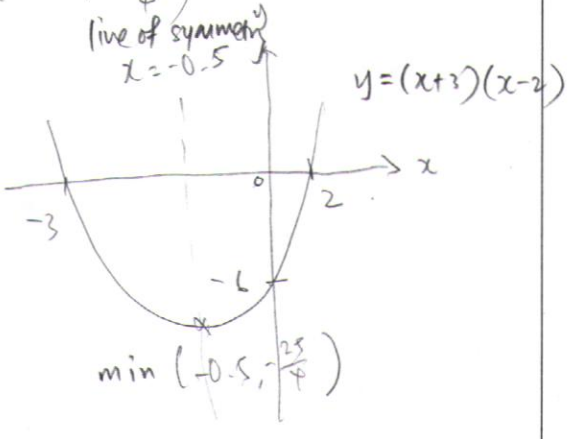
$x=\frac{-3+2}{2}$

$=-0.5$

$y=-\frac{25}{4}$

min $\left(-0.5,-\frac{25}{4}\right)$

line of symmetry $x=-0.5$



c) $y=-(x-1)(x+4)$ $=-(x^2+3x-4)$ $=-x^2-3x+4$

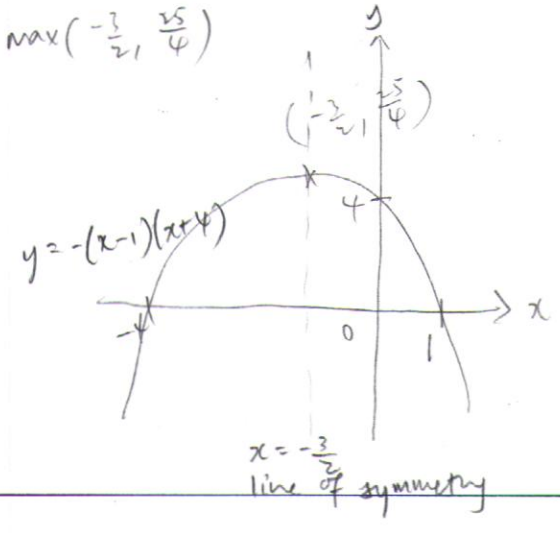
$x=1$ or -4

$x=\frac{1+(-4)}{2}$

$=-\frac{3}{2}$

$y=\frac{25}{4}$

max $\left(-\frac{3}{2},\frac{25}{4}\right)$



line of symmetry $x=-\frac{3}{2}$

d) $y=-(x+5)(x+8)$

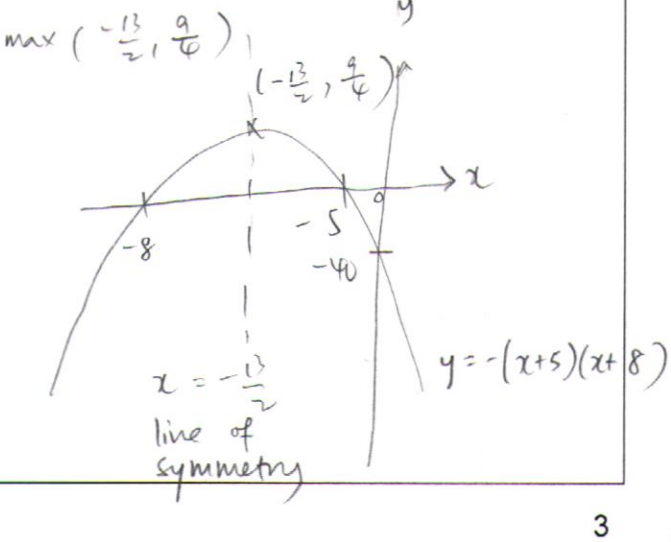
$x=-5$ or -8

$x=\frac{-5+(-8)}{2}$

$=-\frac{13}{2}$

$y=\frac{9}{4}$

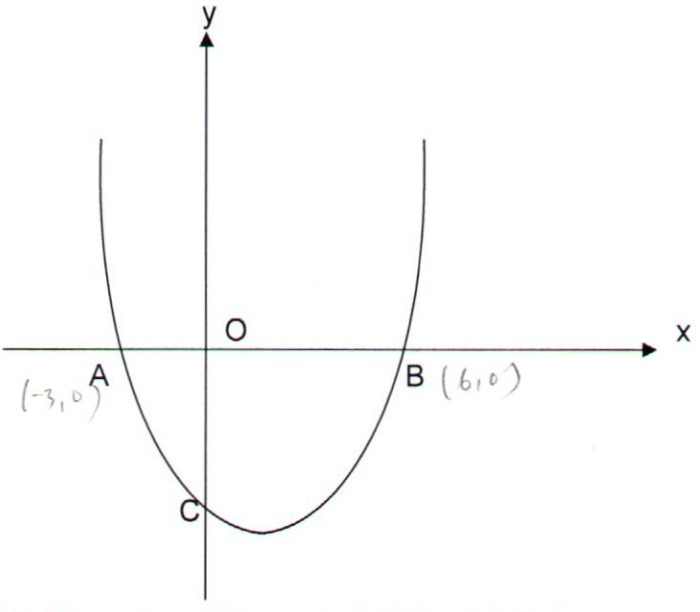
max $\left(-\frac{13}{2},\frac{9}{4}\right)$



line of symmetry $x=-\frac{13}{2}$

2. The curve $y = ax^2 + bx + c$ crosses the axes at A (-3, 0), B (6, 0) and C.

a) Find the values of *a*, *b* and *c*.
b) Hence, state the coordinates of C.
c) The point (5, h) lies on the curve. Find the value of *h*.



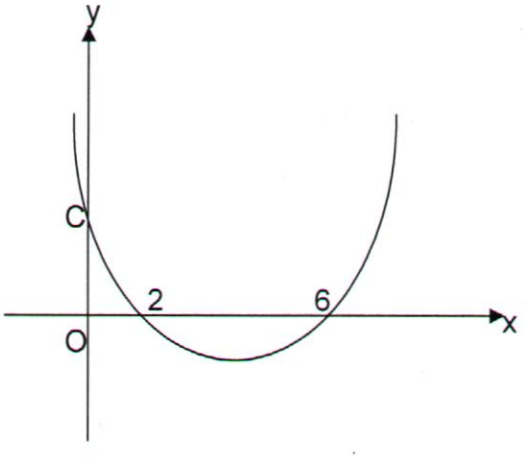
a) $y = (x+3)(x-6)$
$= x^2 - 3x - 18$
$\therefore a = 1, b = -3, c = -18$

b) C (0, -18)

c) when $x = 5$
$y = (5)^2 - 3(5) - 18$
$= -8$
$h = -8$ //

3. The diagram shows the graph of $y = x^2 + ax + b$. The graph cuts the x-axis at (2,0) and (6,0). It cuts the y-axis at C.

a) Calculate the values of *a* and *b*,
b) Write down the coordinates of C.
c) Write down the equation of the line of symmetry of the graph.



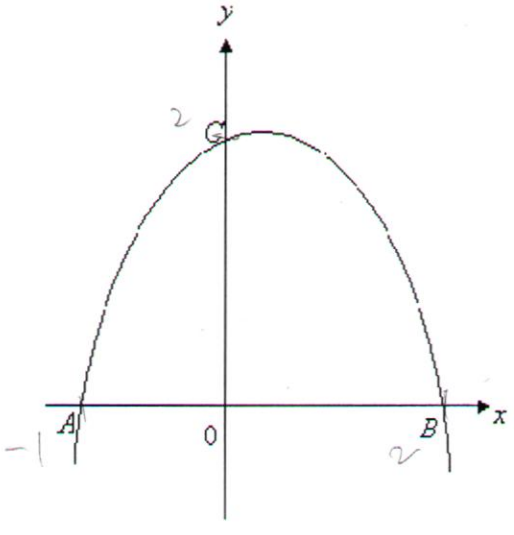
a) $y = (x-2)(x-6)$
$= x^2 - 8x + 12$
$a = -8, \quad b = 12$

b) C (0, 12)

c) $x = 4$ //

4. The graph $y = (x+1)(2-x)$ cuts the *x* axis at *A* and *B*. It cuts the *y*-axis at *C*. Find

(a) the coordinates of the point *C*,
(b) the length *AB*,
(c) the equation of the line of symmetry,
(d) the maximum value of *y*.



a) C (0, 2)

b) AB = 3

c) $x = \frac{-1+2}{2}$
$= \frac{1}{2}$
$x = \frac{1}{2}$

(d) max value of y
= 2.25

Answers:
2a) a=1, b=-3, c=-18 2b) (0, -18) 2c) h=-8
3a) a=-8, b=12 3b) (0, 12) 3c) x=4
3a) (0, 2) (b) 3 units (c) x= 0.5 (d) 2.25

Homework

1. Sketch the graphs for the following quadratic equations, showing clearly:
   - x-intercept
   - y-intercept
   - line of symmetry (state the equation)
   - max/min point (state the coordinates)

a) $y = -x(x+4)$

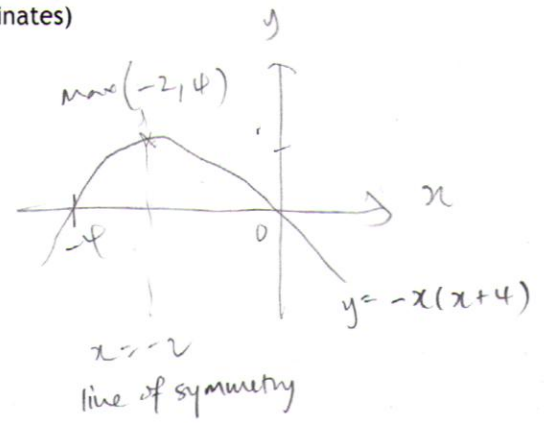
$x = 0$ or -4

$x = \frac{0-4}{2}$

$= -2$

$y = 4$

max $(-2, 4)$



b) $y = x^2 - 6x + 8$

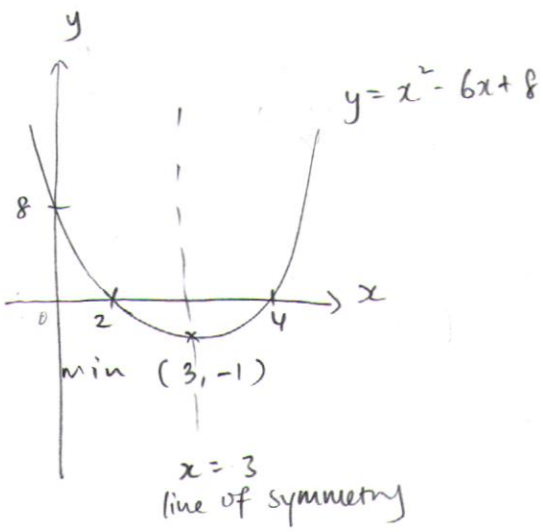
$= (x-2)(x-4)$

$x = \frac{2+4}{2}$

$= 3$

$y = -1$

min $(3, -1)$



c) $y = (x+2)(x-1)$

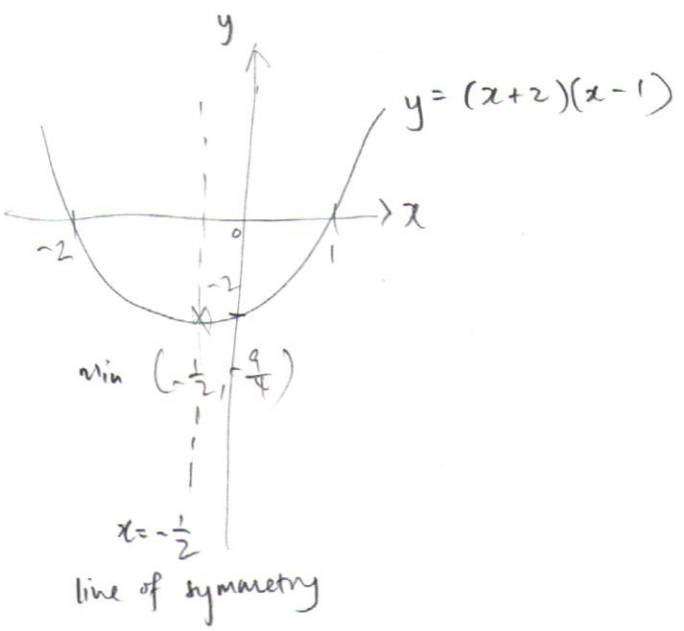
$x = -2, 1$

$x = \frac{(-2)+(1)}{2}$

$= -\frac{1}{2}$

$y = -\frac{9}{4}$

min $(-\frac{1}{2}, -\frac{9}{4})$



d) $y = -x^2 + 60x$

$= -(x)(x-60)$

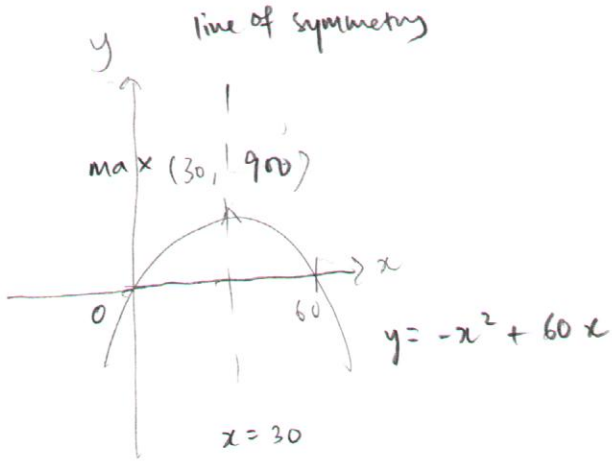
$x = 0$ or 60

$x = \frac{0+60}{2}$

$= 30$

$y = 900$

max $(30, 900)$



2. A graph of $y = (x+2)(6-x)$ cuts the y-axis at A and the x-axis at B and C.
   a) Find the coordinates of A,
   b) Find the coordinates of B and C,
   c) The equation of the line of symmetry
   d) Sketch the graph, showing clearly all of the above.

(a)

when $x = 0$,

$y = 12$

$A(0, 12)$

b) $x = -2$, or $x = 6$

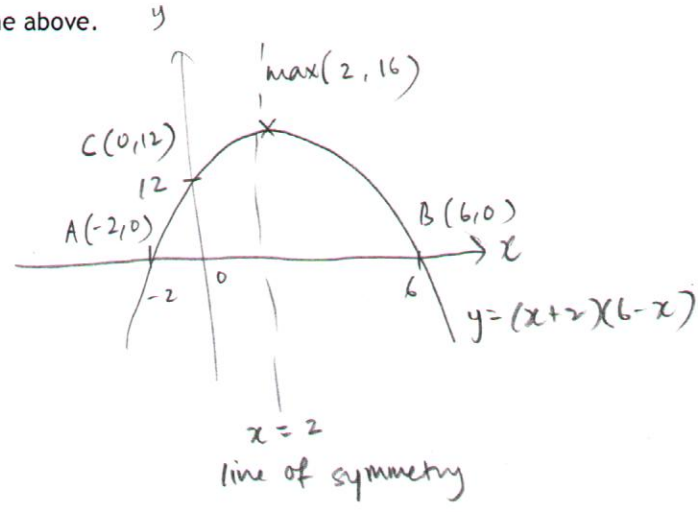
$B(-2, 0)$, $C = (6, 0)$

c) $x = \frac{-2+6}{2}$

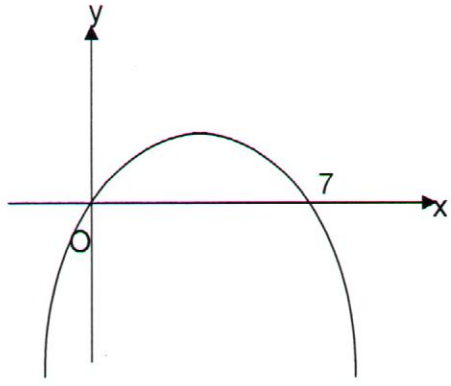
$= 2$

line of symmetry is $x = 2$

d)



3. The sketch represents the graph of the parabola $y = -x^2 + bx + c$. Find
   a) the values of *b* and *c*
   b) the coordinates of the turning point of the parabola.



a) $y = -(x)(x-7)$

$= -x^2 + 7x$

$b = 7$, $c = 0$

b) $x = \frac{0+7}{2}$

$= 3.5$

$y = -(3.5)^2 + 7(3.5)$

$= 12.25$

$= \frac{49}{4}$

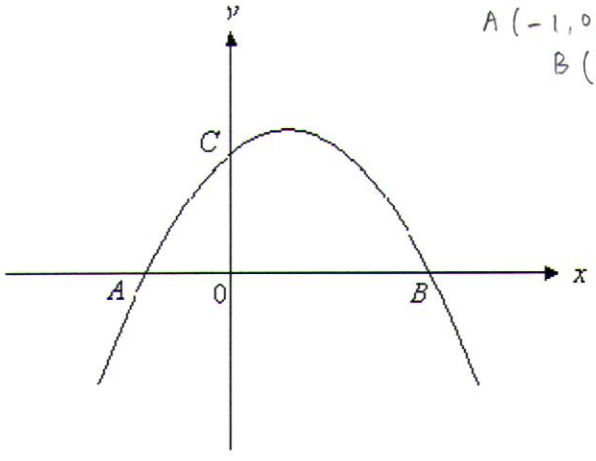
Turning point $= (\frac{7}{2}, \frac{49}{4})$

7. The diagram shows part of the graph of $y = (x+1)(2-x)$, where *A* and *B* lie on the *x*-axis and *C* lies on the *y*-axis.

$x = -1$ or 2

$A(-1, 0)$

$B(2, 0)$



when $x = 0$,

$y = 2$

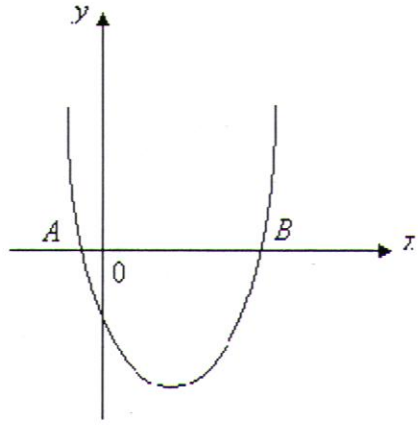
$C(0, 2)$

8. The diagram shows a quadratic curve $y = (2x+3)(2x-9)$. The curve intersects the *x*-axis at *A* and *B*.

(a) Write down the coordinates of *A* and *B*.

(b) State the equation of the line of symmetry of the curve.

(c) Find the coordinates of the minimum point on the curve.



(a) $2x + 3 = 0$ or $2x - 9 = 0$

$x = -\frac{3}{2}$ or $x = \frac{9}{2}$

$A(-\frac{3}{2}, 0)$, $B(\frac{9}{2}, 0)$

b) $x = \frac{-\frac{3}{2} + \frac{9}{2}}{2}$

$= \frac{3}{2}$

line of symmetry is $x = \frac{3}{2}$

c) $y = -36$

min $(\frac{3}{2}, -36)$

Answers:

2a) A (0, 12) 2b) B (-2,0); C(6,0) 2c) x=2

3a) b=7, c=0 3b) (3.5, 12.25) 4a) A (7,0); B (1, 0); C (0, 7)

5a) $-(x - 2)^2 + 1$ 5c) (2, 1) 6a) (-5, 0) 6b) (2,0) 6c) (0, 10)

7. (a) $A(-1, 0), B(2, 0), C(2, 0)$ (b) $\left(\frac{1}{2}, 2\frac{1}{4}\right)$ (c) $x = \frac{1}{2}$

8. a) A(-1.5,0); B(4.5,0) 8b) x = 1.5 8c) (1.5, -36)

My Reflection:

Deadline: Apr 2011

4. The graph of $y=(x-4)^2-9$ cuts the x-axis at points A and B. It cuts the y-axis at point C.

a) Find the coordinates of points A, B and C.

b) Find the coordinates of the maximum/minimum point.

c) Sketch the graph, showing the above clearly.

$0 = (x-4)^2 - 9$

$(x-4)^2 = 9$

$x-4 = \pm\sqrt{9}$

$x = -\sqrt{9}+4$ or $x = \sqrt{9}+4$

$x = 1$ or $x = 7$

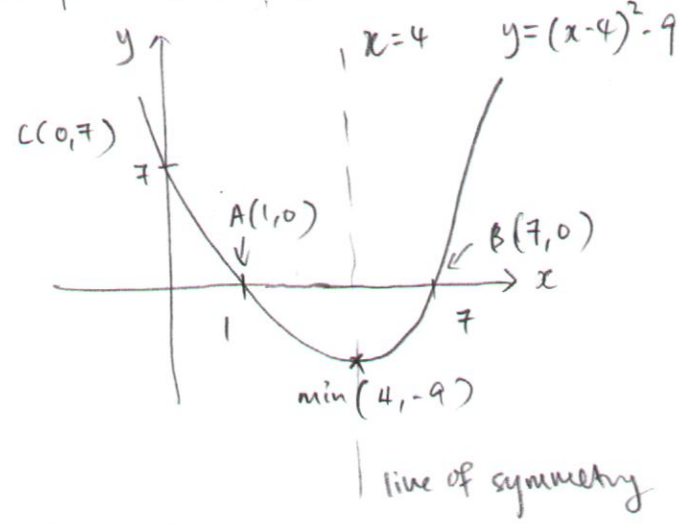
y-intercept : When $x=0$,

$y = 7$

$\therefore$ A(1,0), B(7,0), C(0,7)

b) Min. point = (4, −9)

c)



5. a) Express the equation $-x^2+4x-3$ in the form of $-(x-h)^2+k$ where *h* and *k* are constants.

b) Hence, sketch the graph of $y=-x^2+4x-3$.

c) State the maximum point of the graph.

a) $-x^2+4x-3$

$= -(x^2-4x+3)$

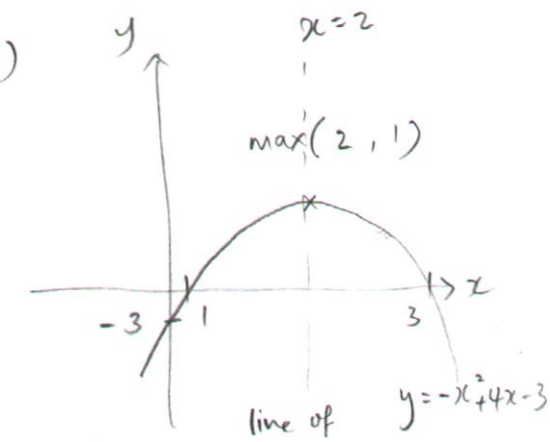
$= -\left(x^2-4x+\left(-\frac{4}{2}\right)^2-\left(-\frac{4}{2}\right)^2+3\right)$

$= -[(x-2)^2-1]$

$= -(x-2)^2+1$

x intercepts : $x=3$ or $x=1$

b)



c) Max. point = (2, 1)

6. The diagram below shows the graph of $y=(x-2)(x+5)$. Points *A*, *B* and *C* lie on the curve. Find the coordinates of

(a) *A*,

(b) *B*,

(c) *C*.

a) A(−5, 0)

b) B(2, 0)

c) C(0, −10)