West Spring Secondary School
2011 3E EM Assignment 2: Functions & Graphs

Index No:

Name: _____ ( ) Class: _____ Date: _____

Overview

This worksheet covers the following:

1. **Recognizing shape of quadratic functions**
2. **Sketching quadratic functions of the form** $y=(x-h)^2+k$ **or** $y=-(x-h)^2+k$

Recap

Previously we learned how to sketch graphs of the form $y=(x-h)^2+k$ **or** $y=-(x-h)^2+k$

But what happens if the quadratic function given is not in the form of $y=(x-h)^2+k$ **or** $y=-(x-h)^2+k$?

Solution: Use completing the square method to change the function.

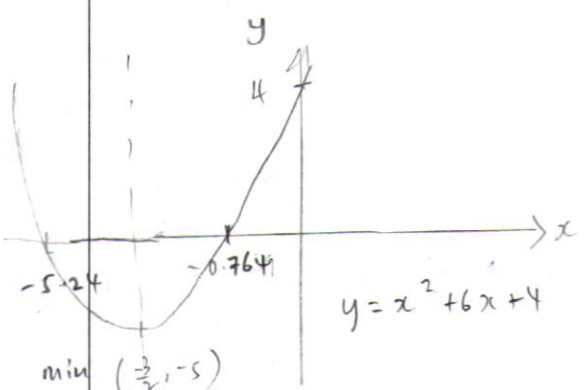
Example/Practice

1. Complete the square for the following quadratic function, then sketch the graph.

a) $y=x^2+6x+4$

$y = x^2+6x+\left(\frac{6}{2}\right)^2-\left(\frac{6}{2}\right)^2+4$

$= (x+\frac{3}{2})^2-5$



$x=-\frac{3}{2}$

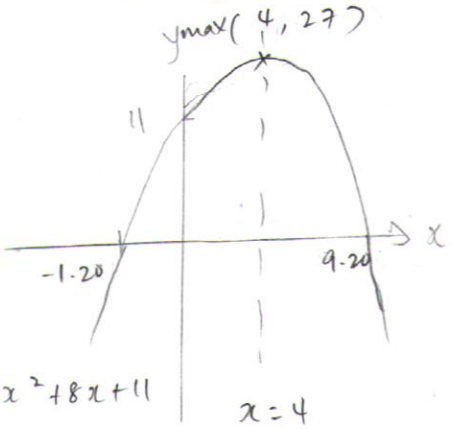
b) $y=-x^2+8x+11$

$= -(x^2-8x-11)$

$= -[x^2-8x+\left(\frac{-8}{2}\right)^2-\left(\frac{-8}{2}\right)^2-11]$

$= -[(x-4)^2-27]$

$= -(x-4)^2+27$



$x=4$

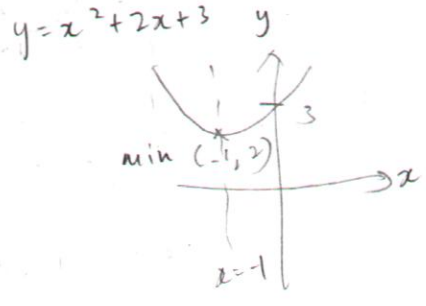
Practice:

1. For each of the following, express the function in the form $y=(x-h)^2+k$ or $y=-(x-h)^2+k$.
   Sketch the graphs, showing clearly:
   - min or max point
   - line of symmetry (state the equation)
   - y-intercept

a) $y=x^2+2x+3$

$= x^2+2x+\left(\frac{2}{2}\right)^2-\left(\frac{2}{2}\right)^2+3$

$= (x+1)^2+2$



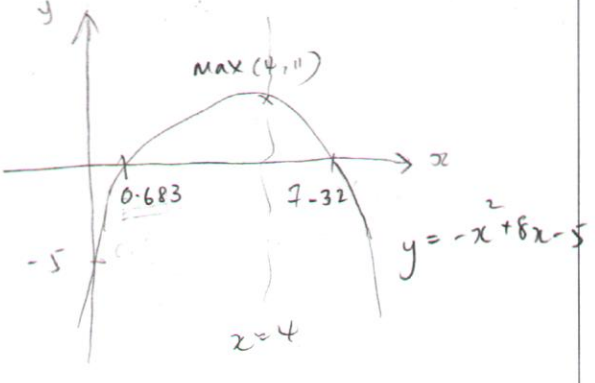
b) $y=-x^2+8x-5$

$= -[x^2-8x+5]$

$= -\left[x^2-8x+\left(\frac{-8}{2}\right)^2-\left(\frac{-8}{2}\right)^2+5\right]$

$= -[(x-4)^2-11]$

$= -(x-4)^2+11$

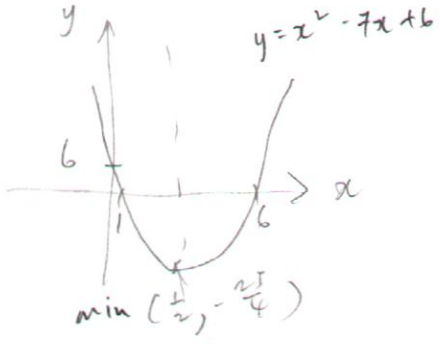


c) $y=x^2-7x+6$

$= x^2-7x+\left(\frac{-7}{2}\right)^2-\left(\frac{-7}{2}\right)^2+6$

$= \left(x-\frac{7}{2}\right)^2-\frac{25}{4}$

min $\left(\frac{7}{2}, -\frac{25}{4}\right)$



$x=\frac{7}{2}$

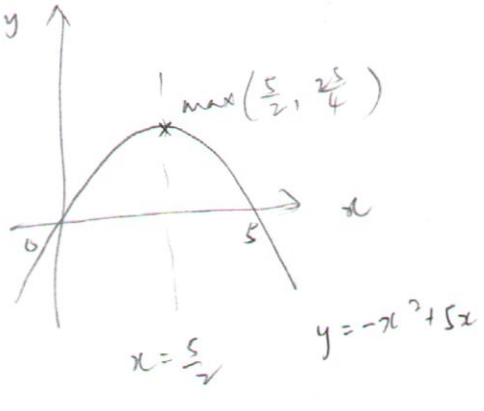
d) $y=-x^2+5x$

$= -\left[x^2-5x+\left(\frac{-5}{2}\right)^2-\left(\frac{-5}{2}\right)^2\right]$

$= -\left[\left(x-\frac{5}{2}\right)^2-\frac{25}{4}\right]$

$= -\left(x-\frac{5}{2}\right)^2+\frac{25}{4}$

max $\left(\frac{5}{2}, \frac{25}{4}\right)$



2. Complete the table below:

Function	Line of symmetry	Turning point	Maximum or minimum point?
$y = x^2$	$x = 0$	(0,0)	Min
$y = -x^2$	$x=0$	$(0,0)$	max
$y = x^2 - 4$	$x=0$	$(0,-4)$	min
$y = (x-3)^2 + 2$	$x=3$	$(3,2)$	min
$y = -(x-4)^2 - 1$	$x=4$	$(4,-1)$	Max
$y = -(x+7)^2$	$x=-7$	$(-7,0)$	~~min~~ max
$y = (x-h)^2 + k$	$x=h$	(h,k)	~~max~~ min
$y = -(x-h)^2 + k$	$x=h$	(h,k)	max

3. The curve $y = a + bx - x^2$ cuts the x-axis at (-3, 0) and (2, 0). Find

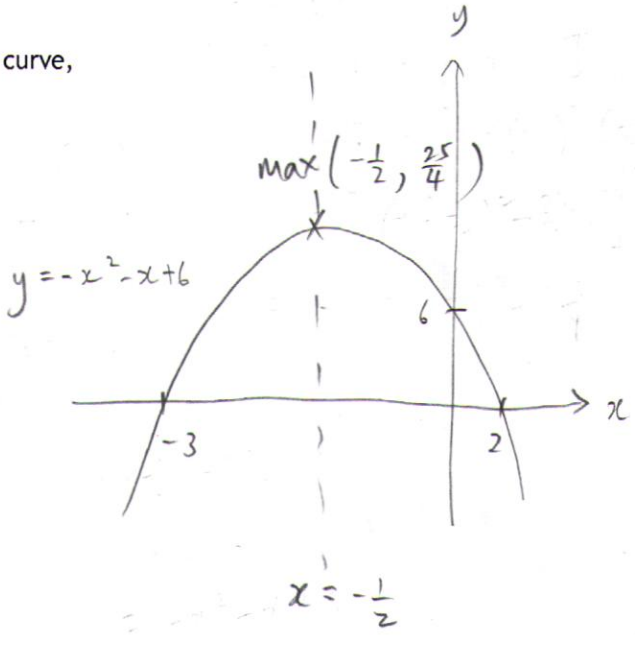
a) Sketch the graph in the space below,
b) the values of a and b,
c) the equation of the line of symmetry of the curve,
d) the value of *k* such that $y \le k$

(b) $y = -(x+3)(x-2)$

$= -[x^2 + x - 6]$

$= -x^2 - x + 6$

$a = 6, \quad b = -1$



(c) $x = \frac{-3+2}{2}$

$= -\frac{1}{2}$

line of symmetry is $x = -\frac{1}{2}$

(d) $y = -(x^2 + x - 6)$

$= -(x^2 + x + (\frac{1}{2})^2 - (\frac{1}{2})^2 - 6)$

$= -(x + \frac{1}{2})^2 + \frac{25}{4}$

$y \le \frac{25}{4}$

Homework

1. For each of the following, express the function in the form $y=(x-h)^2+k$ or $y=-(x-h)^2+k$.
   Sketch the graphs, showing clearly:
   - min or max point
   - line of symmetry (state the equation)
   - y-intercept

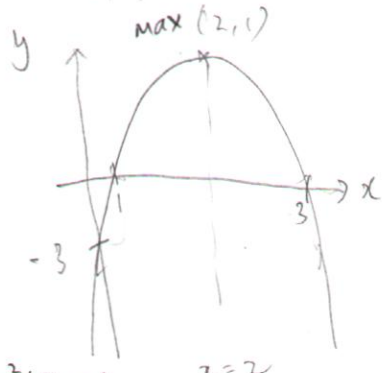
a) $y=-x^2+4x-3$

$=-[x^2-4x+3]$

$=-\left[x^2-4x+\left(\frac{-4}{2}\right)^2-\left(\frac{-4}{2}\right)^2+3\right]$

$=-(x-2)^2+1$

max $(2,1)$



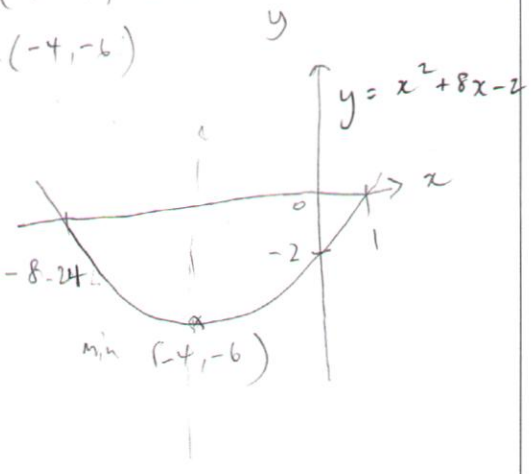
$y=-x^2+4x-3$ $\quad x=2$

b) $y=x^2+8x-2$

$=x^2+8x+\left(\frac{8}{2}\right)^2-\left(\frac{8}{2}\right)^2-2$

$=(x+4)^2-6$

min $(-4,-6)$



$x=-4$

c) $y=-x^2+4x-7$

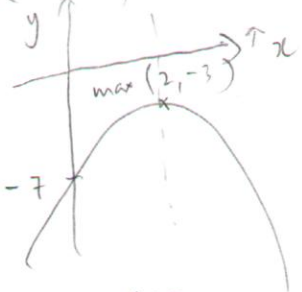
$=-[x^2-4x+7]$

$=-\left[x^2-4x+\left(\frac{-4}{2}\right)^2-\left(\frac{-4}{2}\right)^2+7\right]$

$=-[(x-2)^2+3]$

$=-(x-2)^2-3$

max $(2,-3)$



$x=2$

$y=-x^2+4x-7$

d) $y=-x^2-6x+7$

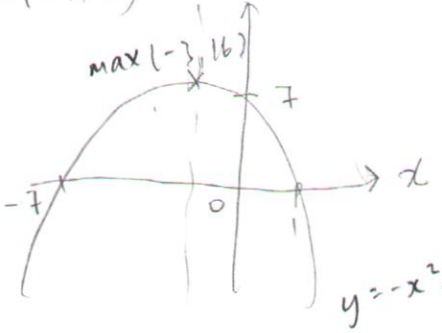
$=-[x^2+6x-7]$

$=-\left[x^2+6x+\left(\frac{6}{2}\right)^2-\left(\frac{6}{2}\right)^2-7\right]$

$=-[(x+3)^2-16]$

$=-(x+3)^2+16$

max $(-3,16)$



$y=-x^2-6x+7$

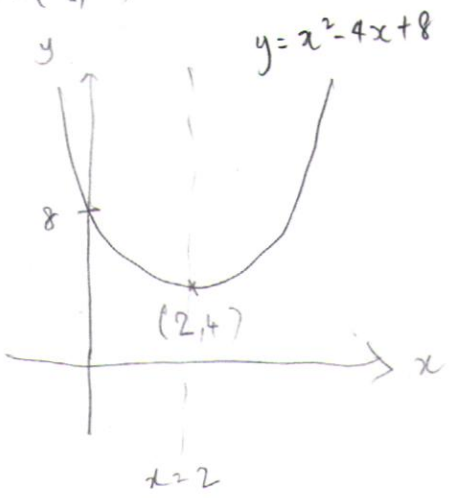
$x=-3$

e) $y = x^2 - 4x + 8$

$= \left[x^2 - 4x + \left(\frac{-4}{2}\right)^2 - \left(\frac{-4}{2}\right)^2 + 8 \right]$

$= (x-2)^2 + 4$

min $(2, 4)$



f) $y = -x^2 - 5x - 2$

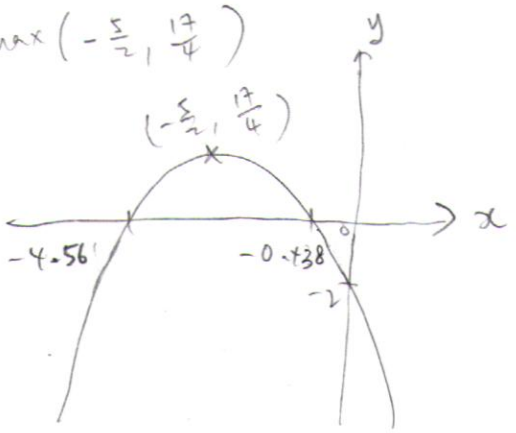
$= -(x^2 + 5x + 2)$

$= -\left(x^2 + 5x + \left(\frac{5}{2}\right)^2 - \left(\frac{5}{2}\right)^2 + 2\right)$

$= -\left[(x + \frac{5}{2})^2 - \frac{17}{4}\right]$

$= -(x + \frac{5}{2})^2 + \frac{17}{4}$

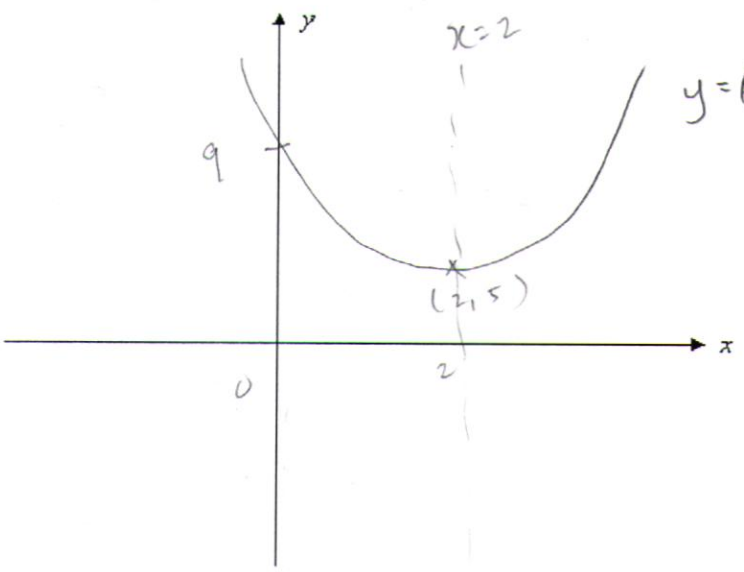
max $\left(-\frac{5}{2}, \frac{17}{4}\right)$



$y = -x^2 - 5x - 2$

$x^2 - 4x + 9$

2. Sketch $y = (x-2)^2 + 5$ in the axes provided below. min $(2, 5)$
(Show the turning point, y-intercept and line of symmetry)



Deadline: Mar 2011