West Spring Secondary School
2011 3E EM Assignment 7: Functions & Graphs

Name: _____ ( ) Class: _____ Date: _____

Overview

This worksheet covers the following:

1. **Gradient of a curve at a point**
2. **Application problems**

Recap

Previously, we learned about graphs for the following functions:

- Linear graphs ✓
- Quadratic graphs ✓
- Cubic graphs ✓
- Reciprocal graphs ✓
- Exponential graphs ✓

Today, we will be learning:

For any graph,
Gradient of a curve at a point = Gradient of the tangent to the curve at the point

What is a tangent to the curve?
A tangent is a straight line that touch a point on the curve.

Definition of Gradient of a curve at a point

We can approximately find the **gradient of a curve at a certain point** by drawing a straight line which touches that point on the curve and then find the gradient of that straight line.

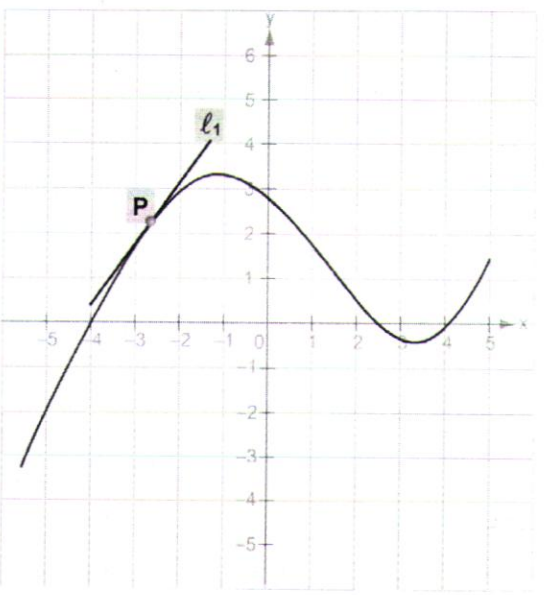
The straight line is called the **tangent** to the curve at the point.

Gradient of curve at a point
= Gradient of tangent drawn to the curve at the point

ℓ_1 **touches** the curve at **P**.

ℓ_1 is called the **tangent to the curve** at **P**.

Gradient of Curve at **P**
= Gradient of Tangent ℓ_1



Practice

1. Find the gradient of the following quadratic curve at points (1, -2), (3, -6) and (4, -5).

Gradient at $(1,-2)$

$= \frac{-4}{1}$

$= -4$ //

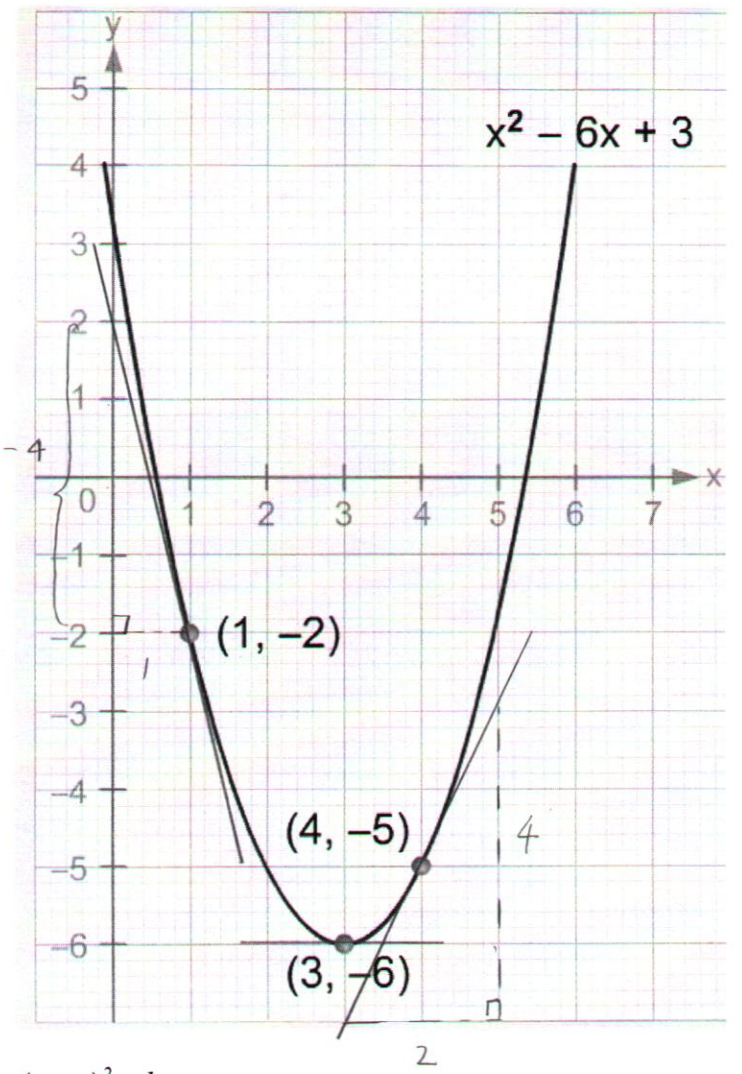
Gradient at $(3,-6)$

$= 0$ //

Gradient at $(4,-5)$

$= \frac{4}{2}$

$= 2$ //



2. (a) Express $x^2 - 2x - 8$ in the form $(x-a)^2 - b$.

(b) Sketch the graph of $y = x^2 - 2x - 8$.

(a) $x^2 - 2x - 8$

$= x^2 - 2x + \left(\frac{-2}{2}\right)^2 - \left(\frac{-2}{2}\right)^2 - 8$

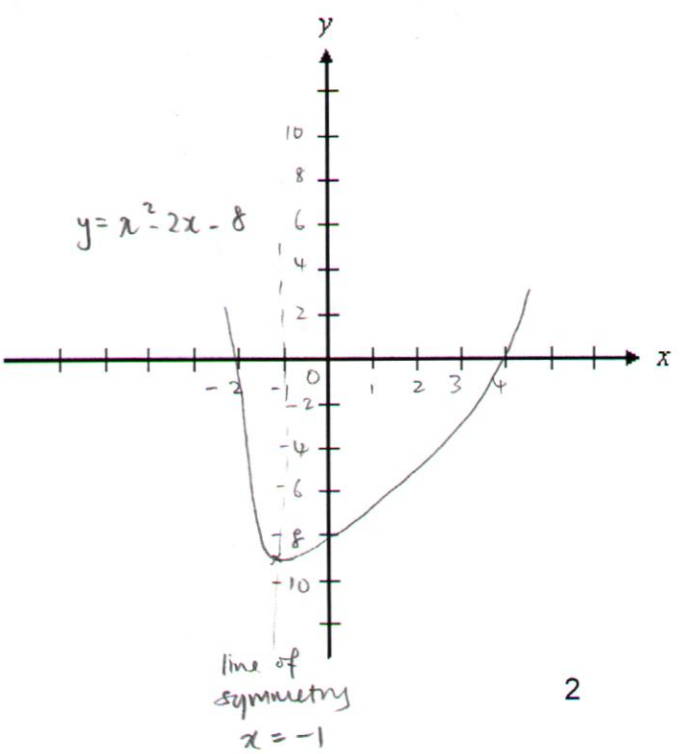
$= (x-1)^2 - 9$ //

(b) min $(-1, -9)$

line of symmetry: $x = -1$

x-intercepts: $x = -2$ or $x = 4$

y-intercept: $y = -8$



Answers:

2. (a) $(x-1)^2 - 9$

Practice 1:

Plot the graph for the function $y = 2x^2 - 3x - 4$ for $-4 \le x \le 5$ by completing the table below.

x	-4	-3	-2	-1	0	1	2	3	4	5
y	40	23	10	1	-4	-5	-2	5	16	31

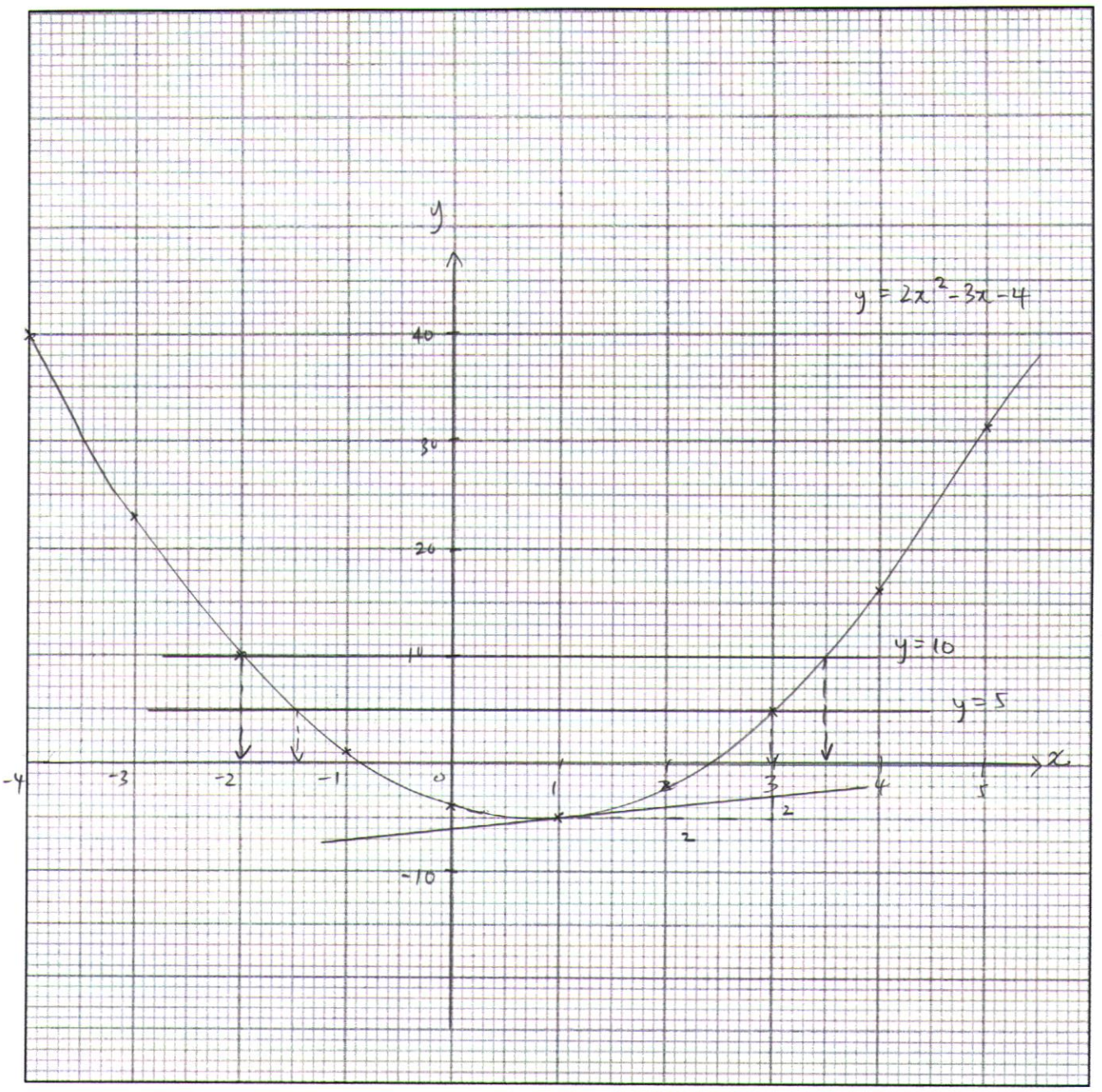
a) Use your graph to solve:

i. $2x^2 - 3x - 4 = 0$

ii. $2x^2 - 3x - 4 = 10$

iii. $2x^2 - 3x - 9 = 0$

b) Find the gradient of the curve at x = 1



a i) $x = 0.9$ or $x = 2.4$

ii) $x = -2$ or $x = 3.5$

iii) $2x^2 - 3x - 9 = 0$

$2x^2 - 3x - 9 + 5 = 5$

$2x^2 - 3x - 4 = 5$

$x = -1.5$ or $x = 3.5$

b) Gradient $= \frac{2}{2}$

$= 1$

2. Answer the whole of this question on a sheet of graph paper.

The number of bacteria, n, in a colony is given by the equation $n = 25(2^t)$ where t is time in hours.

The table below shows the number of bacteria, n, in the colony after time t hours.

t/h	0	1	2	3	4	5
n	25	50	p 100	200	400	800

(a) Find the value of p.

(b) Using a horizontal scale of 2 cm to represent 1 hour and a vertical scale of 2 cm to represent 100 bacteria, draw the graph of $n = 25(2^t)$ for $0 \le t \le 5$.

(c) Use your graph to find

(i) the number of bacteria in the colony after 3.5 hours,

(ii) the time when the number of bacteria is 500.

(d) **(i)** By drawing a tangent, find the gradient of the curve when $t = 3$.

(ii) State briefly what this gradient represents.

(e) The number of bacteria in another colony is given by the equation $n = 600 - 150t$.

(i) On the same axes, draw a graph to represent the number of bacteria in this colony.

(ii) Find the value of t when the numbers in the colonies are equal.

3. A ball is thrown and it flies in a trajectory, which can be represented by a quadratic function, across the field. The ball reaches its highest point of 5 m above a point 20 m away from where it is thrown.

a) How far away will the ball land from where it is thrown?

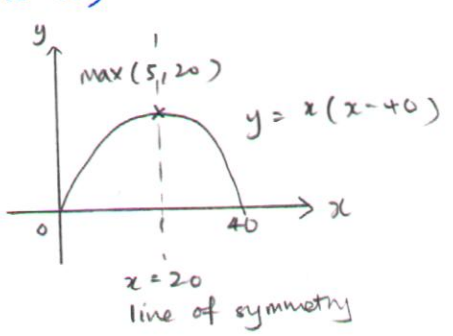
b) What is the quadratic equation for the ball's flight?
Express it in the form $y = a(x-p)(x-q)$ where a, p and q are constants.

c) Sketch the path of the ball
(Assume the ball starts at the point (0, 0).)

(a) Distance $= 20 \times 2$
$= 40$m

(b) $y = x(x-40)$

(c)



Answers:
1i) x=-0.85, 2.35 ii) x=-2, 3.45 iii) x=-1.55, 3 1b) m = 0 2a) p=100 2ci) n=280
2cii) t= 4.4h 2di) m=131.6 2dii) no. of bacteria grown in one hour at that instant 2eii) t=2.85 h
3a) 40m 3b) y= x(x-40)

Homework

1. **Answer the whole of this question on a sheet of graph paper.**

The variables x and y are connected by the equation $y = x + \frac{15}{x} - 6$.

Some corresponding values of x and y, correct to 1 decimal place, are given in the table below.

x	1	1.5	2	3	4	5	6	7	8
y	10	a	3.5	2	1.8	2	b	3.1	3.9

5.5 2.5

(a) Calculate the value of a and of b.

(b) Using a scale of 2 cm to represent 1 unit, draw a horizontal axis for $0 \le x \le 8$.

Using a scale of 2 cm to represent 1 unit, draw a vertical axis for $0 \le y \le 10$.

On these axes, draw the graph of $y = x + \frac{15}{x} - 6$.

(c) Use your graph to find the value of x when the gradient of the curve $y = x + \frac{15}{x} - 6$ is equal to zero.

(d) By drawing a tangent, find the gradient of the graph at $x = 2$.

(e) The line $y = kx$ touches the curve $y = x + \frac{15}{x} - 6$ at exactly one point.

By drawing a suitable straight line on the same axes, use your graph to find the least positive value of k.

(f) By drawing a suitable straight line on the same axes, use your graph to find the solutions of the equation $\frac{3}{2}x + \frac{15}{x} - 12 = 0$.

2. **Answer the whole of this question on a sheet of graph paper.**

When x copies of a book are produced, the cost, \$$y$, of each copy is given by the formula $y = 10 + \frac{3600}{x}$. The table below gives some values of x and the corresponding values of y.

x	100	200	300	400	600	900	1200
y	46	28	22	19	16	14	13

(a) Using a horizontal scale of 2 cm to represent 200 books and a vertical scale of 2 cm to represent \$5, draw a graph of y against x.

(b) Use your graph to estimate the number of books to be printed if the cost of producing each book is \$20.

(c) By drawing a tangent, find the gradient of the curve when $x = 400$.

(d) State briefly what this gradient represents.

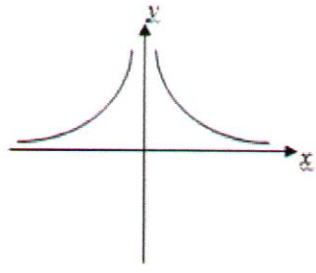
(e) In order to sell x books, the selling price of each book must be \$$(30 - \frac{x}{50})$.

On the same axes, draw the graph of $y = 30 - \frac{x}{50}$ to represent the selling price of the books.

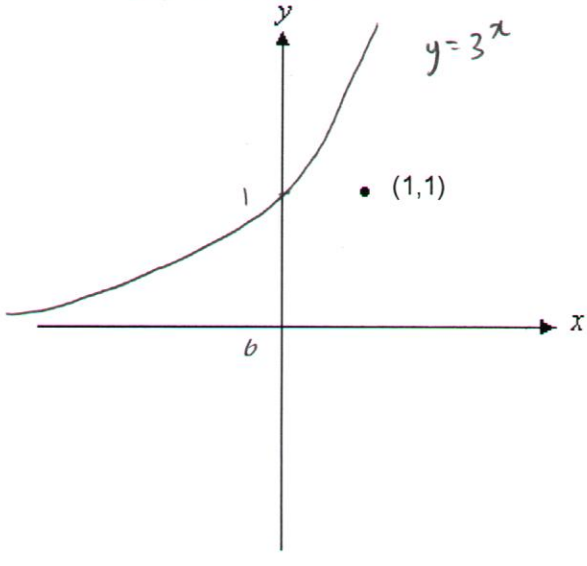
(f) Using your graphs, find the range of the number of books that should be printed if no loss is to be made, assuming all the books will be sold.

3. The equation of the graph shown below are of the form $y = x^n$, where n is an integer. State a possible value of n.

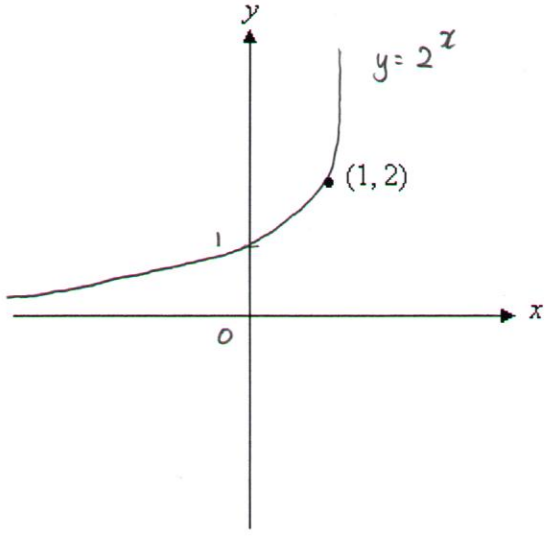
$n = -2$



4. a) The point (1, 1) is marked on the diagram in the answer space. On the diagram, sketch the graphs of $y = 3^x$.



b) The point (1, 2) is marked on the diagram in the answer space. On the diagram, sketch $y = 2^x$.



Answers

1. (a) $a = 5.5$, $b = 2.5$ (c) $x =$ 3.9 6.4 (d) -2.7 (e) 0.40 (f) 1.5 or

2. (b) (360 ± 20) books (c) -0.02 (d) Cost is reduced at the rate of 2 cents per copy. Or The rate of reduction of the cost of production per copy of book.

(f) $240(\pm 20) \le x \le 770(\pm 20)$

(c) Gradient $= \dfrac{26 - 9.5}{100 - 800}$

$= -0.02$

3) n= -2

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Practice 2a

(a) $p = 100$

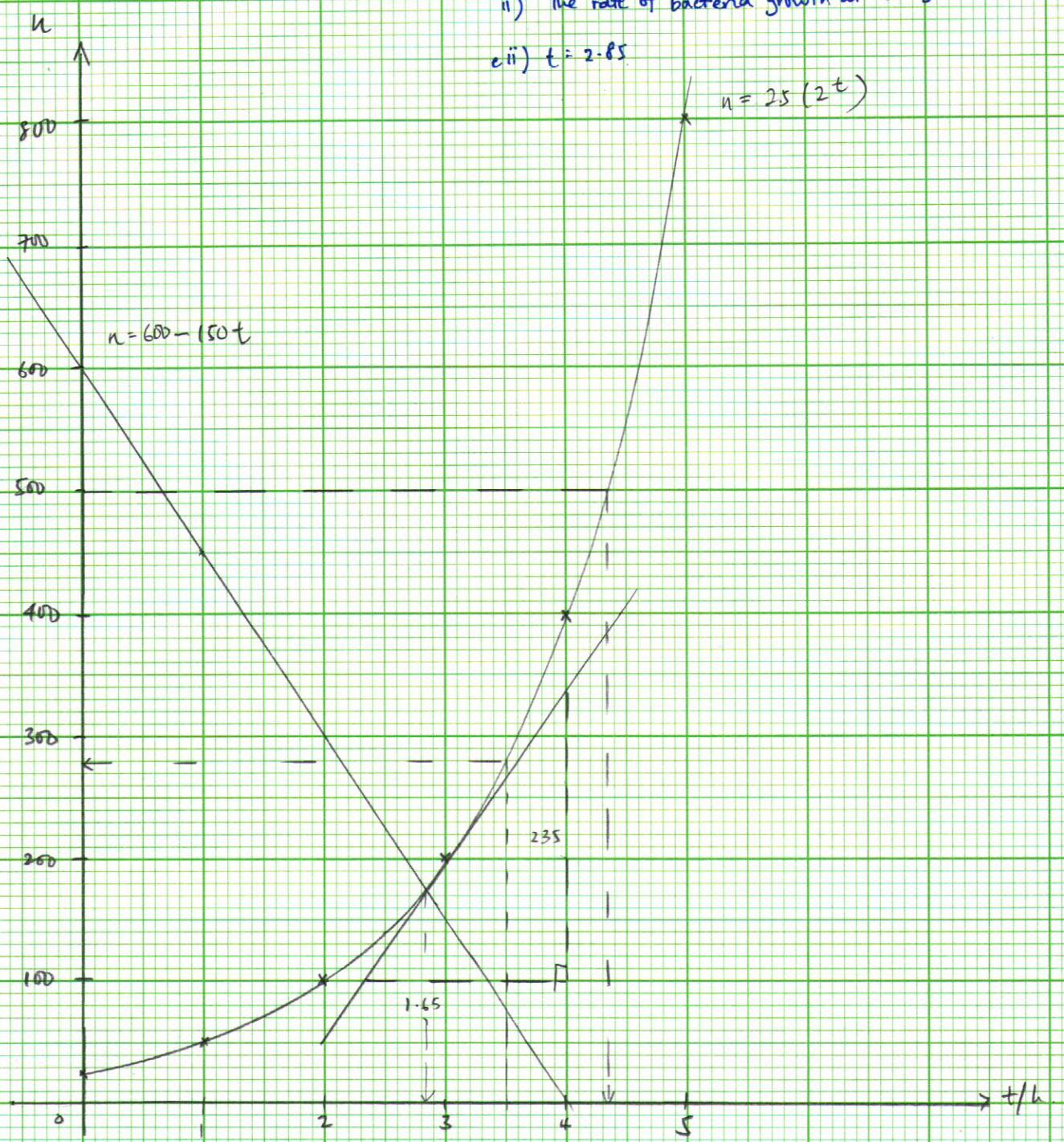
(b) No. of bacteria = 280

(c) (ii) Time = 4.35 h

d i) Gradient = $\frac{235}{1.65}$ = 142. (correct to 3 sig. fig.)

ii) the rate of bacteria growth at $t = 3$

e ii) $t = 2.85$



20 cm x 24 cm

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Homework Qn 1

(a) $a = 5.5$, $b = 2.5$

(c) $x = 3.8$

d) Gradient $= \frac{-2.8}{1}$

$= -2.8$

e) least value of $k = \frac{1}{2.5}$

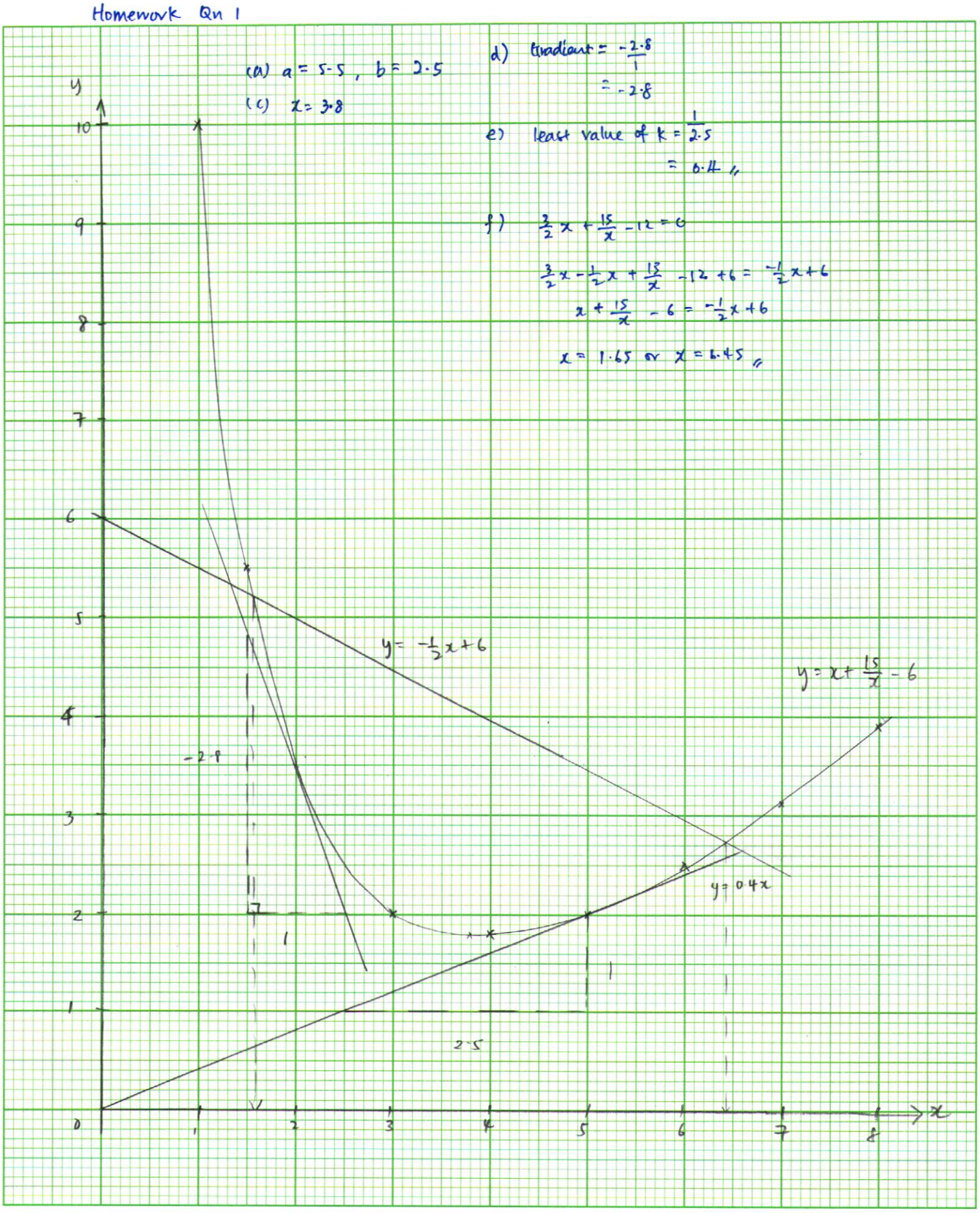
$= 0.4$

f) $\frac{3}{2}x + \frac{15}{x} - 12 = 0$

$\frac{3}{2}x - \frac{1}{2}x + \frac{15}{x} - 12 + 6 = -\frac{1}{2}x + 6$

$x + \frac{15}{x} - 6 = -\frac{1}{2}x + 6$

$x = 1.65$ or $x = 6.45$



20 cm x 24 cm

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Homework Qn 2

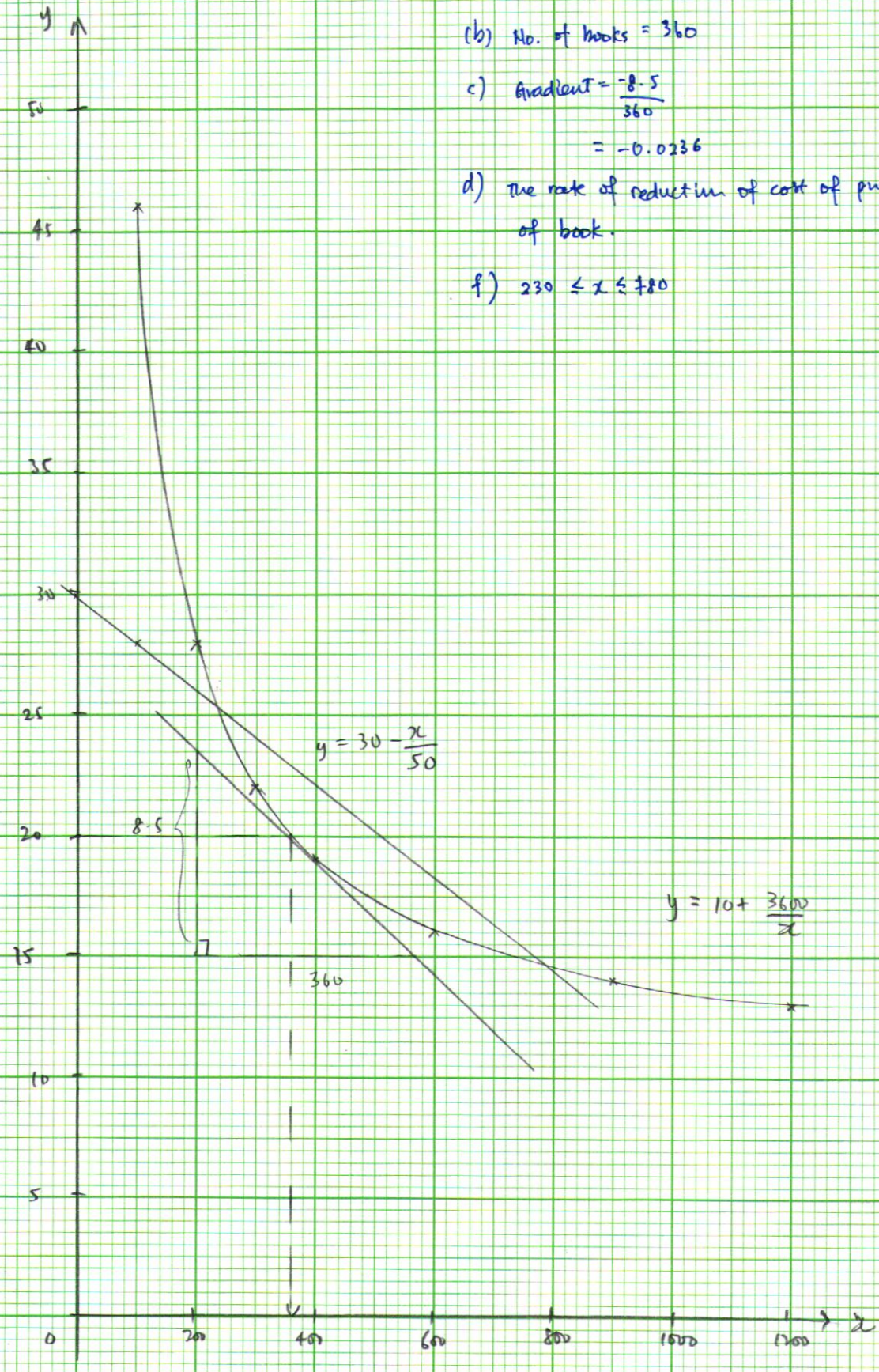
(b) No. of books = 360

c) Gradient $= \frac{-8.5}{360}$

$= -0.0236$

d) The rate of reduction of cost of production per copy of book.

f) $230 \leq x \leq 780$



20 cm x 24 cm