West Spring Secondary School
2011 3E EM Assignment 4: Congruence and Similarity

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Name: ____________________ ( ) **Class:** __________ **Date:** ________

Team Manager: ________________
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Overview

This worksheet covers the following:

1. **Volume of similar figures**

Recap

Previously, we learned about the relationship between areas of similar figures.

Areas of Similar Figures

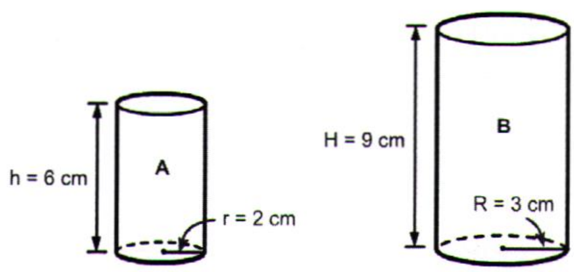
$$\frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2$$

- A_1 and A_2 are the areas of the similar figures
- l_1 and l_2 are their corresponding lengths

The ratio of areas of two similar figures is equal to the square of the ratio of two corresponding lengths of the figures.

In this worksheet, we are going to investigate the relationship between volumes of similar figures.

Example 1:



$$\frac{\text{Height, H of cylinder B}}{\text{Height, h of cylinder A}} = \frac{9}{6} = \frac{3}{2}$$

$$\frac{\text{Radius, R of cylinder B}}{\text{Radius, r of cylinder A}} = \frac{3}{2}$$

Let's find the volume of A and B:

$$\text{Vol. of cylinder A} = \pi r^2 h = \pi(2)^2(6) = 24\pi \text{ cm}^3$$

$$\text{Vol. of cylinder B} = \pi(3)^2(9) = 81\pi \text{ cm}^3$$

$$\frac{\text{Vol. of cylinder B}}{\text{Vol. of cylinder A}} = \frac{81\pi}{24\pi} = \frac{27}{8}$$

$$\Rightarrow \left(\frac{\text{Radius of cylinder B}}{\text{Radius of cylinder A}}\right)^3 = \left(\frac{3}{2}\right)^3 = \frac{27}{8}$$

Volumes of Similar Solids

For any similar solids, the ratio of their volumes is equal to the cube of the ratio of any 2 of their corresponding lengths.

$$\boxed{\frac{V_1}{V_2} = \left(\frac{l_1}{l_2}\right)^3}$$

- V_1 and V_2 are the volumes of the similar solids
- l_1 and l_2 are their corresponding lengths

Examples:

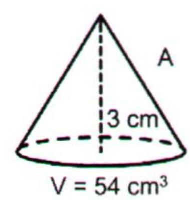
1. 2 similar cones, A and B, have heights 3 cm and 4 cm respectively. If cone A is 54 cm³, find the volume of cone B.

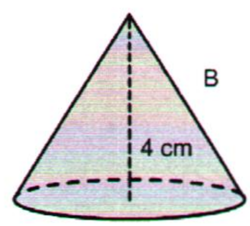
$$\frac{\text{Vol. of cone B}}{\text{Vol. of cone A}} = \left(\frac{l_2}{l_1}\right)^3$$

$$\frac{\text{Vol. of cone B}}{54} = \left(\frac{4}{3}\right)^3$$

$$\therefore \text{Vol. of cone B} = \frac{64}{27} \times 54$$

$$= 128 \text{ cm}^3$$





2. Pyramids A and B are similar. The volume of A is 12 cm³ and the volume of B is 324 cm³. Find the height of A if the height of B is 18 cm.

$$\frac{\text{Vol. of A}}{\text{Vol. of B}} = \left(\frac{l_1}{l_2}\right)^3$$

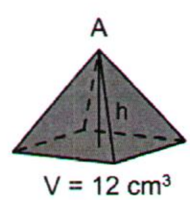
$$\frac{12}{324} = \left(\frac{h_1}{18}\right)^3$$

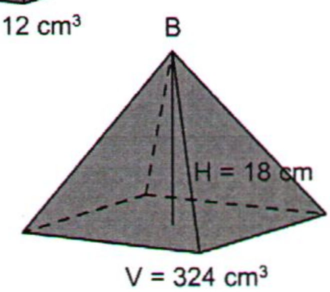
$$\frac{12}{324} = \frac{h_1^3}{5832}$$

$$h_1^3 = 216$$

$$h_1 = \sqrt[3]{216}$$

$$\therefore \text{Height of A} = 6 \text{ cm}$$





3. The heights of 2 geometrically similar cylinders are 36 cm and 9 cm respectively.

a) If the diameter of the base of the larger cylinder is 5 cm, find the diameter of the base of the smaller cylinder.

b) If the base area of the smaller cylinder is 48 cm^2, calculate the base area of the larger cylinder.

a)
$$\frac{\text{Height of cylinder A}}{\text{Height of cylinder B}} = \frac{9}{36} = \frac{1}{4}$$
$$\frac{\text{Diameter of cylinder A}}{\text{Diameter of cylinder B}} = \frac{1}{4}$$
$$\frac{\text{Diameter of cylinder A}}{5} = \frac{1}{4}$$
Diameter of cylinder A = 1.25 cm

b)
$$\frac{\text{Base Area of cylinder A}}{\text{Base Area of cylinder B}} = \left(\frac{l_1}{l_2}\right)^2$$
$$\frac{48}{\text{Base Area of cylinder B}} = \left(\frac{1}{4}\right)^2$$
$$\frac{\text{Base Area of cylinder B}}{48} = \frac{16}{1}$$
Base Area of cylinder B = 768 cm^2

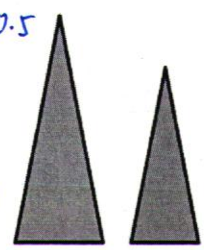
Answers:

1) 128 cm^3 2) 6 cm 3a) 1.25 cm 3b) 768 cm^2

Practice

1. The weights of 2 similar marble statues are 729 g and 343 g. If one statue is 3 cm higher than the other, what is the height of the shorter statue?

Let h_1 be height of shorter statue.
$$\frac{\text{Vol. of Statue A}}{\text{Vol. of Statue B}} = \left(\frac{h_1}{h_2}\right)^3$$
$$\frac{343}{729} = \left(\frac{h_1}{h_1+3}\right)^3$$
$$\sqrt[3]{\frac{343}{729}} = \frac{h_1}{h_1+3}$$
$$\frac{7}{9} = \frac{h_1}{h_1+3}$$
$$7(h_1+3) = 9h_1$$
$$7h_1 + 21 = 9h_1$$
$$2h_1 = 21$$
$$h_1 = 10.5$$
∴ Height of shorter statue is 10.5 cm



2. The surface area of 2 spheres are 72 cm^2 and 50 cm^2.
Find the ratio of their: a) radii
b) volumes

(a)
$$\frac{\text{Surface Area of Sphere A}}{\text{Surface Area of Sphere B}} = \left(\frac{l_1}{l_2}\right)^2 = \frac{72}{50} = \frac{36}{25}$$
∴ Ratio of radii = $\sqrt{36} : \sqrt{25}$
$= 6 : 5$

(b) Ratio of volume = $6^3 : 5^3$
$= 216 : 125$

3. Two similar bowls have capacities 240 cm^3 and 810 cm^3 respectively.
   a) Find the ratio of the depth of the smaller bowl to that of the larger bowl
   b) If the base area of the larger bowl is 72 cm^2, find the base area of the smaller bowl.

(a)

$$\frac{\text{Vol. of Bowl A}}{\text{Vol. of Bowl B}} = \left(\frac{l_1}{l_2}\right)^3$$

$$\frac{240}{810} = \left(\frac{l_1}{l_2}\right)^3$$

$$\frac{8}{27} = \left(\frac{l_1}{l_2}\right)^3$$

$$\text{Ratio of depth} = \sqrt[3]{8} : \sqrt[3]{27}$$

$$= 2 : 3 \;,,$$

b)

$$\frac{\text{Base Area of smaller bowl}}{\text{Base Area of larger bowl}} = \left(\frac{2}{3}\right)^2$$

$$\frac{\text{Base Area of smaller bowl}}{72} = \frac{4}{9}$$

$$\text{Base Area of smaller bowl} = \frac{4}{9} \times 72$$

$$= 32 \text{ cm}^2 \;,,$$

Answers:

1) 10.5 cm 2a) 6:5 2b) 216:125 3a) 2:3 3b) 32 cm^2

Homework

1. Two plant pots are geometrically similar. The height of the smaller pot is 5 cm. The height of the larger pot is 15 cm.
   a) The diameter of the base of the larger pot is 7 cm. Find the diameter of the base of the smaller pot.
   b) Find the ratio of the volume of the smaller pot to that of the larger.

(a)

$$\frac{h_1}{h_2} = \frac{5}{15} = \frac{1}{3}$$

$$\frac{\text{Diameter of smaller pot}}{\text{Diameter of larger pot}} = \frac{1}{3}$$

$$\frac{\text{Diameter of smaller pot}}{7} = \frac{1}{3}$$

$$\text{Diameter of smaller pot} = \frac{1}{3} \times 7$$

$$= 2\tfrac{1}{3} \text{ cm} \;,,$$

b)

$$\frac{\text{Vol. of smaller pot}}{\text{Vol. of larger pot}} = \left(\frac{1}{3}\right)^3$$

$$= \frac{1}{27}$$

$$\therefore \text{Ratio} = 1 : 27 \;,,$$

2. The **areas** of the bases of two similar jugs are in the ratio 4 : 9.
   a) Find the ratio of the height of the smaller jug to the height of the larger jug.
   b) Find the ratio of volume of smaller jug to volume of larger jug.
   c) Given that the volume of the larger jug is 405 cm^3, find the volume of the smaller jug.

(a) Ratio of height

$$= \sqrt{4} : \sqrt{9}$$

$$= 2 : 3 \;,,$$

(b) Ratio of volume

$$= (2)^3 : (3)^3$$

$$= 8 : 27 \;,,$$

c)

$$\frac{\text{Vol. of smaller jug}}{\text{Vol. of larger jug}} = \frac{8}{27}$$

$$\frac{\text{Vol. of smaller jug}}{405} = \frac{8}{27}$$

$$\text{Vol. of smaller jug} = 120 \text{ cm}^3 \;,,$$

3. The areas of the bases of two similar toys are in the ratio 4 : 9.
   a) Find the ratio of their heights.
   b) Find the ratio of the volume of the smaller toy to that of the larger toy.
   c) If the volume of the larger toy is 81 cm^3, find the volume of the smaller toy.

a) Ratio of heights

$= \sqrt{4} : \sqrt{9}$

$= 2 : 3$ //

b) Ratio of volume

$= (2)^3 : (3)^3$

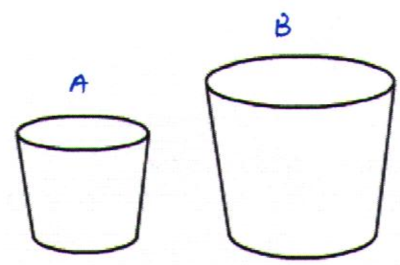
$= 8 : 27$ //

c) $\frac{\text{Vol. of smaller toy}}{\text{Vol. of larger toy}} = \frac{8}{27}$

$\frac{\text{Vol. of smaller toy}}{81} = \frac{8}{27}$

Vol. of smaller toy $= 24 \text{ cm}^3$ //

4. The ratio of the areas of the bases of two geometrically similar buckets is 16 : 25.
   a) Write down the ratio of the heights of the two buckets.
   b) Find the ratio of their volumes.
   c) Both buckets are filled with sand. The mass of sand in the larger bucket is 55 kg. Find the mass of sand in the smaller bucket.



a) Ratio of heights

$= \sqrt{16} : \sqrt{25}$

$= 4 : 5$ //

b) Ratio of volumes

$= 4^3 : 5^3$

$= 64 : 125$ //

c) $\frac{\text{Mass of sand in A}}{\text{Mass of sand in B}} = \frac{64}{125}$

$\frac{\text{Mass of sand in A}}{55} = \frac{64}{125}$

Mass of sand in smaller bucket $= 28.16$ kg //

5. The base area of an actual tower is 612 000 cm^2. The model of this tower has a base area of 1700 cm^2.
   a) Find the ratio of the height of the actual tower to that of the model of the tower.
   b) If the height of the actual tower is 90 m, calculate the height of the model of the tower.
   c) Given that the volume of the model is 10 000 cm^3, find the volume of the actual tower.

(a) Ratio of heights

$= \sqrt{\frac{612000}{1700}}$

$= \sqrt{\frac{360}{1}}$

$= 18.97 : 1$

$= 19 : 1$ //

b) $\frac{\text{Height of model}}{\text{Height of actual tower}} = \frac{1}{19}$

$\frac{\text{Height of model}}{90} = \frac{1}{19}$

Height of model $= 4.736$

$= 4.74$ m (correct to 3 sig. fig.)

c) $\frac{\text{Vol. of actual tower}}{\text{Vol. of model}} = \left(\frac{19}{1}\right)^3$

$\frac{\text{Vol. of actual tower}}{10\,000} = \frac{6859}{1}$

Vol. of actual tower $= 6859 \times 10000$

$= 68590000 \text{ cm}^3$ //

6. A round cake of radius 10 cm has a height of 5 cm. A similar but bigger cake has a radius of 15 cm. Taking $\pi = 3.142$, find:
   a) the height of the big cake
   b) the volume of the big cake (in litres, correct to 4 sig. fig.) (1 litre = 1000 cm^3)

(a) $\frac{R_1}{R_2} = \frac{10}{15} = \frac{2}{3}$

$\frac{H_1}{H_2} = \frac{2}{3}$

$\frac{5}{H_2} = \frac{2}{3}$

$\frac{H_2}{5} = \frac{3}{2}$

$H_2 = 7.5$ cm

Height of big cake is 7.5 cm

b) Vol. of Big Cake

$= 3.142(15)^2(7.5)$

$= 5302.125\ cm^3$

$= 5.3021$

$= 5.302$ l (correct to 4 sig. fig.)

7. The volume of Cylinder A is 1024 cm^3. A similar cylinder, B has base radius <u>thrice</u> that of Cylinder A. Another cylinder C, whose base area is 576 cm^2, has ¼ the height and three times the radius of cylinder A. Find
   a) the volume of cylinder B,
   b) the volume of cylinder C,
   c) the base area of cylinder A.

(a) $\frac{R_1}{R_2} = \frac{1}{3}$

$\frac{V_1}{V_2} = \left(\frac{1}{3}\right)^3 = \frac{1}{27}$

$\frac{\text{Vol. of cylinder A}}{\text{Vol. of cylinder B}} = \frac{1}{27}$

$\frac{1024}{\text{Vol. of cylinder B}} = \frac{1}{27}$

$\frac{\text{Vol. of cylinder B}}{1024} = \frac{27}{1}$

Vol. of cylinder B $= 27 \times 1024 = 27648\ cm^3$

(b) $\frac{\text{Vol. of Cylinder A}}{\text{Vol. of cylinder C}} = \frac{\pi r^2 h}{\pi (3r)^2(\frac{1}{4}h)}$

$\frac{1024}{\text{Vol. of cylinder C}} = \frac{r^2 h}{9r^2(\frac{1}{4}h)}$

$\frac{1024}{\text{Vol. of cylinder C}} = \frac{r^2 h}{\frac{9}{4} r^2 h}$

$\frac{1024}{\text{Vol. of cylinder C}} = \frac{4}{9}$

Vol. of cylinder C $= 2304\ cm^3$

c) Base Area of cylinder A $= 1024 \div 16 = 64\ cm^2$

Answers:
1a) 2.33 cm 1b) 1:27
2a) 2:3 2b) 8:27 2c) 120 cm^3
3a) 2:3 3b) 8:27 3c) 24 cm^3
4a) 4:5 4b) 64:125 4c) 28.16 kg
5a) 19:1 5b) 4.74m 5c) 68,590,000 cm^3
6a) 7.5cm 6b) 5.302 litres
7a) 27,648 cm^3 7b) 2304 cm^3 7c) 64 cm^2

Summary

Volumes of Similar Solids

For any similar solids, the ratio of their volumes is equal to the cube of the ratio of any 2 of their corresponding lengths.

$$\boxed{\frac{V_1}{V_2} = \left(\frac{l_1}{l_2}\right)^3}$$

- V_1 and V_2 are the volumes of the similar solids
- l_1 and l_2 are their corresponding lengths