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West Spring Secondary School
2011 3E EM Assignment 3: Congruence and Similarity

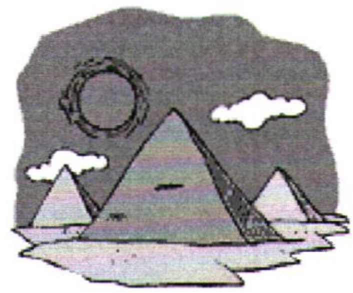
Name: ____________________ ( ) **Class:** __________ **Date:** __________

Team Manager: ____________________
Row no: ________

Overview

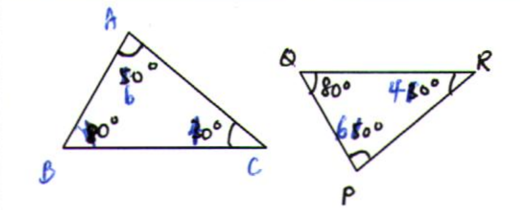
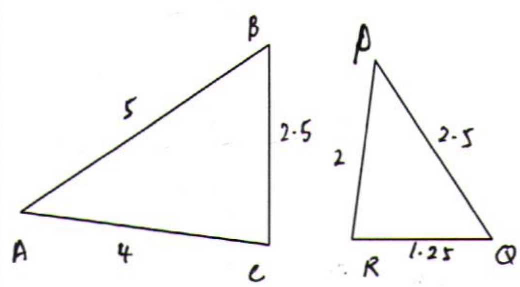
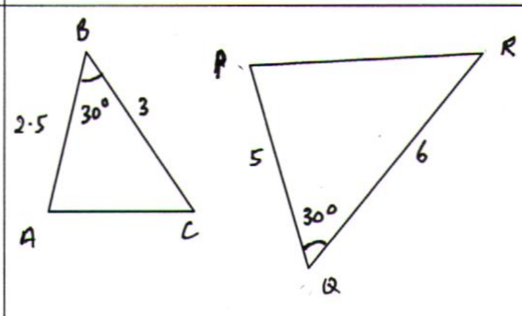
This worksheet covers the following:

1. **Application on Similar Triangles**
2. **Area of similar figures**



Recap

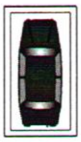
Previously, we learned about similarity and the 3 tests for similiar triangles.
Complete the table below to refresh your memory.

Test	Description	Diagram	
AAA	All 3 corresponding angles are equal		$\angle ABC =$ ∠PQR $\angle BCA =$ ∠QRP $\angle CAB =$ ∠RPQ ΔABC is similar to ΔPQR (AAA).
SSS	All 3 corresponding sides are in equal proportion		$\frac{AB}{PQ} = \frac{5}{2.5} = 2$ $\frac{BC}{QR} = \frac{2.5}{1.25} = 2$ $\frac{CA}{RP} = \frac{4}{2} = 2$ ΔABC is similar to ΔPQR (SSS).
SAS	2 corresponding sides are in equal proportion and 1 corresponding included angle is equal		$\frac{AB}{PQ} = \frac{2.5}{5} = \frac{1}{2}$ $\angle ABC = \angle PQR = 30°$ $\frac{BC}{QR} = \frac{3}{6} = \frac{1}{2}$ ΔABC is similar to ΔPQR (SAS).

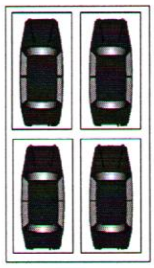
In this worksheet, we are going to investigate the relationship between areas of similar figures.

Example 1:

Consider a car park, which can accommodate a single car

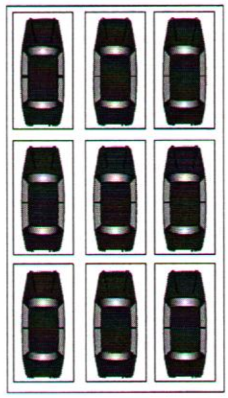


Observe what happens if we double the sides of the car park



It can accommodate 4 times as many cars

If we treble the sides of the car park...



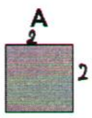
It can accommodate 9 times as many cars

As the car park is doubled, what happens to its area?

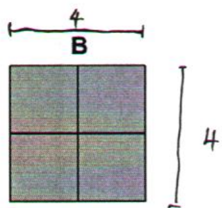
When it's trebled?

Example 2:

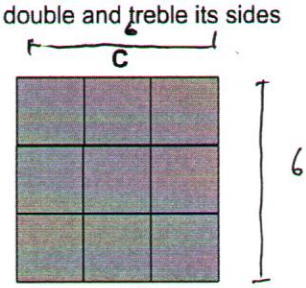
Consider a square A of side 2 cm. Notice what happens to the area when you double and treble its sides



Area = 4 cm^2



Area = 16 cm^2



Area = 36 cm^2

Ratio of sides of A and B = 2 : 4 = 1 : 2

Ratio of areas of A and B = 4 : 16 = 1 : 4

Relationship? $(l_1)^2 : (l_2)^2$

Ratio of areas of A and B = $\left(\frac{l_1}{l_2}\right)^2$

Ratio of sides of A and C = 4 : 6 = 2 : 3

Ratio of areas of A and C = 16 : 36 = 4 : 9

Areas of Similar Figures

The ratio of the areas of any two similar figures is equal to the square of the ratio of any two corresponding lengths of the figures.

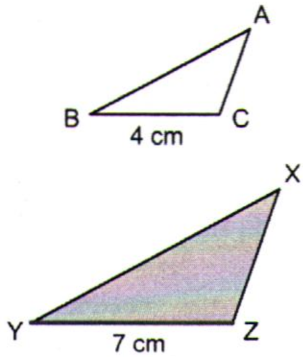
$$\frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2$$

- A_1 and A_2 are the areas of the similar figures
- l_1 and l_2 are their corresponding lengths

Example:

1.

ΔABC and ΔXYZ are similar triangles. Given that BC is 4 cm, YZ is 7 cm, and the area of ΔXYZ is 98 cm^2, find the area of ΔABC.



$$\frac{BC}{YZ} = \frac{4}{7}$$

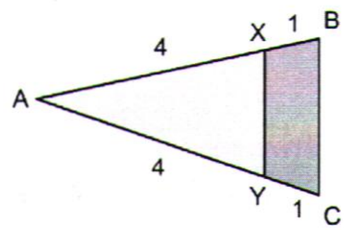
$$\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta XYZ} = \left(\frac{4}{7}\right)^2 = \frac{16}{49}$$

$$\frac{\text{Area of } \Delta ABC}{98} = \frac{16}{49}$$

$$\therefore \text{ Area of } \Delta ABC = \frac{16}{49} \times 98 = 32 \text{ cm}^2$$

2. In the diagram, $\dfrac{AX}{XB} = \dfrac{AY}{YC} = \dfrac{4}{1}$.

If the area of ΔAXY is 48 cm^2, find the area of XYCB.



$$\frac{\text{Area of } \Delta AXY}{\text{Area of } \Delta ABC} = \left(\frac{4}{5}\right)^2 = \frac{16}{25}$$

$$\frac{48}{\text{Area of } \Delta ABC} = \frac{16}{25}$$

$$\frac{\text{Area of } \Delta ABC}{48} = \frac{25}{16}$$

$$\text{Area of } \Delta ABC = \frac{25}{16} \times 48 = 75 \text{ cm}^2$$

$$\therefore \text{ Area of } XYCB = 75 - 48 = 27 \text{ cm}^2$$

Answers:
1. 32 cm^2 2. 27 cm^2

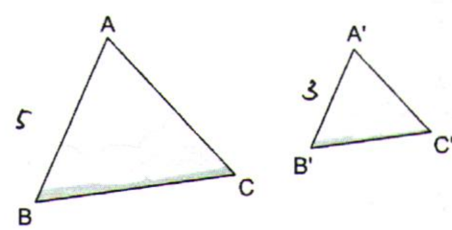
Practice

1.

Triangle ABC and triangle A'B'C' are similar.

$\dfrac{AB}{A'B'} = \dfrac{5}{3}$ and the area of triangle ABC = 45 cm^2

Find the area of A'B'C'



$$\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta A'B'C'} = \left(\frac{5}{3}\right)^2$$

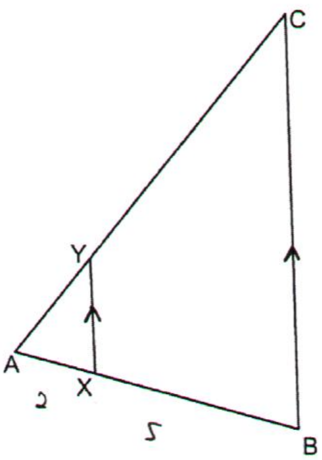
$$\frac{45}{\text{Area of } \Delta A'B'C'} = \frac{25}{9}$$

$$\frac{\text{Area of } \Delta A'B'C'}{45} = \frac{9}{25}$$

$$\text{Area of } \Delta A'B'C' = \frac{9}{25} \times 45 = 16.2 \text{ cm}^2$$

2. **ABC is a triangle in which XY || BC.**

5AX = 2XB and area of $\triangle AXY = 16\ cm^2$. Find the area of quadrilateral XYCB.



$5AX = 2XB$

$\frac{AX}{XB} = \frac{2}{5}$ $\quad \therefore \frac{AX}{AB} = \frac{2}{7}$

$\frac{\text{Area of } \Delta AXY}{\text{Area of } \Delta ABC} = \left(\frac{2}{7}\right)^2$

$\frac{16}{\text{Area of } \Delta ABC} = \frac{4}{49}$

$\frac{\text{Area of } \Delta ABC}{16} = \frac{49}{4}$

$\text{Area of } \Delta ABC = \frac{49}{4} \times 16$

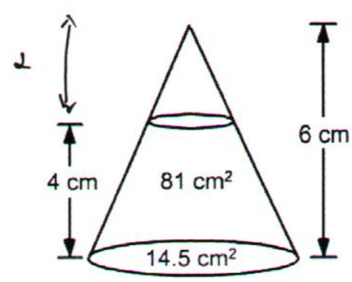
$= 196\ cm^2$

$\therefore$ Area of quadrilateral XYCB

$= 196 - 16$

$= 180\ cm^2$ //

3. A 6 cm tall cone is filled with water to a depth of 4 cm. If the internal surface area of its wall is 81 cm², and its base is 14.5 cm², find its surface are which is wet.



Curved SA $= 81\ cm^2$

$\frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2$

$\frac{A_1}{81} = \left(\frac{2}{6}\right)^2$

$\frac{A_1}{81} = \left(\frac{1}{3}\right)^2$

$\frac{A_1}{81} = \frac{1}{9}$

$A_1 = 9\ cm^2$

Water Curved Surface Area $= 81 - 9$

$= 72\ cm^2$

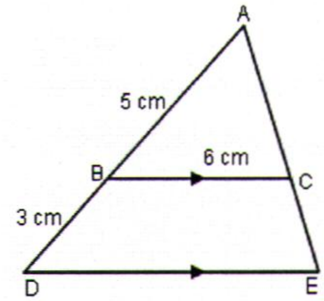
Surface Area Wet $= 72 + 14.5$

$= 86.5\ cm^2$ //

Answers:
1. 16.2 cm² 2. 180 cm² 3. 86.5 cm²

Homework

1. In the diagram ABD and ACE are straight lines and BC is parallel to DE. AB = 5 cm, BC = 6 cm and BD = 3 cm.
   a) Find the length DE.
   b) The area of the triangle ABC is 20 cm^2. Calculate the area of triangle ADE.



(a) $\frac{AB}{AD} = \frac{5}{8}$

$\frac{BC}{DE} = \frac{5}{8}$

$\frac{6}{DE} = \frac{5}{8}$

$\frac{DE}{6} = \frac{8}{5}$

$DE = \frac{8}{5} \times 6$

$= 9.6$ cm

(b) $\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta ADE} = \left(\frac{5}{8}\right)^2$

$\frac{20}{\text{Area of } \Delta ADE} = \frac{25}{64}$

$\frac{\text{Area of } \Delta ADE}{20} = \frac{64}{25}$

$\text{Area of } \Delta ADE = \frac{64}{25} \times 20$

$= 51.2$ cm^2

2. The surface areas of two similar cylinders are 50 cm^2 and 32 cm^2
   a) Find the ratio of the diameters of their circular bases.
   b) If the height of the bigger cylinder is 10 cm, find the height of the smaller cylinder.

(a) $\frac{\text{Diameter of } C_1}{\text{Diameter of } C_2} = \sqrt{\frac{50}{32}}$

$\frac{\text{Diameter of } C_1}{\text{Diameter of } C_2} = \sqrt{\frac{25}{16}}$

$= \frac{5}{4}$

$\therefore$ Ratio of diameters $= 5 : 4$

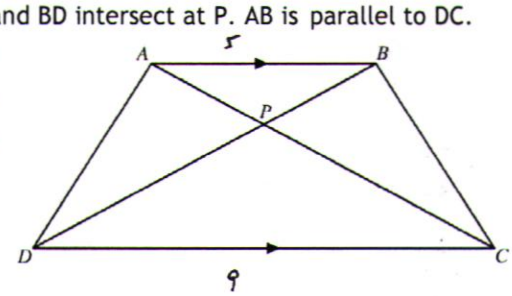
(b) $\frac{h_1}{h_2} = \frac{5}{4}$

$\frac{10}{h_2} = \frac{5}{4}$

$\frac{h_2}{10} = \frac{4}{5}$

$h_2 = \frac{4}{5} \times 10 = 8$ cm

3. In the quadrilateral shown below, the diagonals AC and BD intersect at P. AB is parallel to DC.
   a) Name a triangle which is similar to ΔAPB. Show the similarity by explaining the reasons clearly.
   b) Given that AB = 5 cm, CD = 9 cm and the area of ΔAPB = 75 cm^2, find the area of the triangle that is similar to ΔAPB in (a).



(a) ΔAPB is similar to ΔCPD

$\angle APB = \angle CPD$ (vert. opp. $\angle$s)

$\angle BAP = \angle DCP$ (alt. $\angle$s, AB // DC)

$\angle ABP = \angle CDP$ (alt. $\angle$s, AB // CD)

$\therefore \Delta APB$ is similar to ΔCPD (AAA similarity)

(b) $\frac{\text{Area of } \Delta APB}{\text{Area of } \Delta CPD} = \left(\frac{5}{9}\right)^2$

$\frac{75}{\text{Area of } \Delta CPD} = \frac{25}{81}$

$\frac{\text{Area of } \Delta CPD}{75} = \frac{81}{25}$

$\text{Area of } \Delta CPD = \frac{81}{25} \times 75$

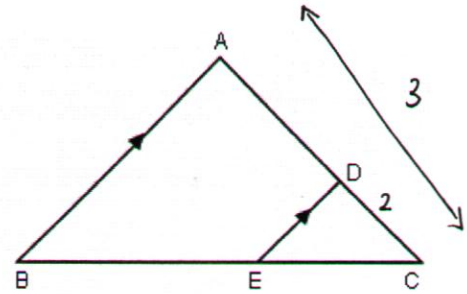
$= 243$ cm^2

4. In the diagram, AB is parallel to DE and $\frac{AC}{DC} = \frac{3}{2}$.

a) Name a pair of similar triangles and prove their similarity.

b) Find the ratio $\frac{area\ of\ \Delta ABC}{area\ of\ \Delta DEC}$.

c) Given that area of ΔDEC is 16 cm^2, find the area of the trapezium ABED.



(a) ΔABC is similar to ΔDEC

$\angle ABC = \angle DEC$ (corr. $\angle$s, AB // DE)

$\angle BAC = \angle EDC$ (corr. $\angle$s, AB // DE)

$\angle ACB = \angle DCE$ (common)

$\therefore$ ΔABC is similar to ΔDEC (AAA similarity)

(b) $\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta DEC} = \left(\frac{3}{2}\right)^2$

$= \frac{9}{4}$

$\therefore$ Ratio $= 9 : 4$ //

5. In the diagram above, AB = 3 cm, BC = 6 cm and AF = 2 cm

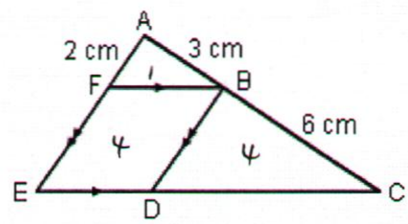
a) Find the length FE

Given that ΔABF and ΔBCD are similar, find the ratio of

b) area of ΔABF : area of ΔBCD

c) area of ΔABF : area of ΔACE

d) area of ΔBCD : area of quadrilateral ABDE



(a) $\frac{AB}{BC} = \frac{3}{6} = \frac{1}{2}$

$\frac{AF}{BD} = \frac{1}{2}$

$\frac{2}{BD} = \frac{1}{2}$

$\frac{BD}{2} = \frac{2}{1}$

$BD = 4$

$FE = BD = 4$ cm //

(b) Ratio $\frac{\text{Area of } \Delta ABF}{\text{Area of } \Delta BCD} = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

$\therefore$ ratio $= 1 : 4$ //

(c) $\frac{\text{Area of } \Delta ABF}{\text{Area of } \Delta ACE} = \left(\frac{3}{9}\right)^2$

$= \frac{1}{9}$

$\therefore$ ratio $= 1 : 9$ //

(d) Area of ΔBCD : Area of ABDE

4 : 5 //

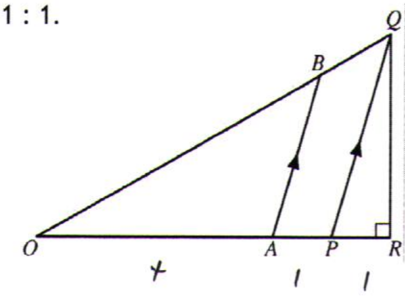
6. In the given figure, AB is parallel to PQ and OA : AP : PR is 4 : 1 : 1.

a) Calculate the value of $\frac{\text{area of } \Delta OAB}{\text{area of } \Delta OPQ}$.

b) If the area of Δ OPQ = 100 cm^2, what is the

i. Area of quadrilateral ABQP?

ii. Area of quadrilateral ABQR?



(a) $\frac{\text{Area of } \Delta OAB}{\text{Area of } \Delta OPQ} = \left(\frac{4}{5}\right)^2$

$= \frac{16}{25}$ //

(b) (i) $\frac{\text{Area of } \Delta OAB}{100} = \frac{16}{25}$

Area of ΔOAB = 64 cm^2

Area of quadrilateral ABQP = 100 − 64

= 36 cm^2 //

$\frac{\text{Area of } ORQ}{\text{Area of } OPQ} = \frac{\frac{1}{2} \times OR \times QR}{\frac{1}{2} \times OP \times QR}$

$= \frac{6}{5}$

(ii)

5 units —— 100 cm^2

6 units —— 120 cm^2

∴ Area of $\Delta ORQ = 120\ cm^2$

∴ Area of ABQR = 120 − 64

= 56 cm^2 //

7. ABCD is a rectangle and M is a point on CD. AC and BM meet at X.

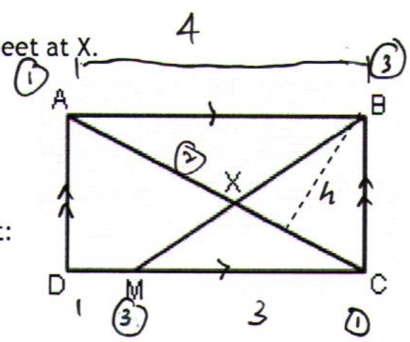
a) Prove that triangles CXM and AXB are similar.

b) It is given that CM = 3MD. Find the ratio of

i. area of Δ CXM : area of Δ AXB

ii. CX : AX

iii. area of Δ BXC : area of rectangle ABCD. (Hint: Δ BXC and Δ ABC share a common height)



(a) $\angle AXB = \angle CXM$ (vert. opp. ∡s)

$\angle BAX = \angle MCX$ (alt. ∡s, AB // CD)

$\angle ABX = \angle CMX$ (alt. ∡s, AB // CD)

∴ ΔCXM is similar to ΔAXB (AAA similarity)

(b) (i) CM = 3MD

$\frac{CM}{MD} = \frac{3}{1}$

$\frac{CM}{AB} = \frac{3}{4}$

Area of ΔCXM : Area of ΔAXB

$(3)^2$: $(4)^2$

9 : 16 //

b ii) CX : AX

3 : 4 //

iii) Let the common height be h.

$\frac{\text{Area of } \Delta BXC}{\text{Area of } ABCD} = \frac{\frac{1}{2} \times h \times XC}{2 \times \frac{1}{2} \times h \times AC}$

$= \frac{XC}{2AC}$

$= \frac{3}{2(7)}$

$= \frac{3}{14}$

∴ ratio = 3 : 14 //

Answers:

1a) 9.6 cm b) 51.2 cm^2 2a) 5:4 b) 8 cm 3b) 243 cm^2 4b) 9:4 4c) 20 cm^2
5a) 4 cm b) 1:4 c) 1:9 d) 4:5 6a) 16:25 bi) 36 cm^2 bii) 56 cm^2
7bi) 9:16bii) 3:4 biii) 3:14

Summary

Areas of Similar Figures

The ratio of the areas of any two similar figures is equal to the square of the ratio of any two corresponding lengths of the figures.

$$\boxed{\frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2}$$

- A_1 and A_2 are the areas of the similar figures
- l_1 and l_2 are their corresponding lengths

My Reflection:

On a range of 0 to 100% (0% - completely don't understand, 100% - understand completely), give a rating for your level of understanding for this lesson.

Provide details on the things / questions / concepts you don't understand.